

MATHEMATICAL STUDY OF THE TRANSFER OF PHYSICAL PROPERTIES IN NEWTONIAN FLUID FLOW BY NUMERICAL METHODS

A Thesis

Submitted towards the Requirements for the Award of Degree of

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IN
MATHEMATICS**

Under the Faculty of Science

Submitted By

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(Enrollment No. 220101072)

Under the Supervision of

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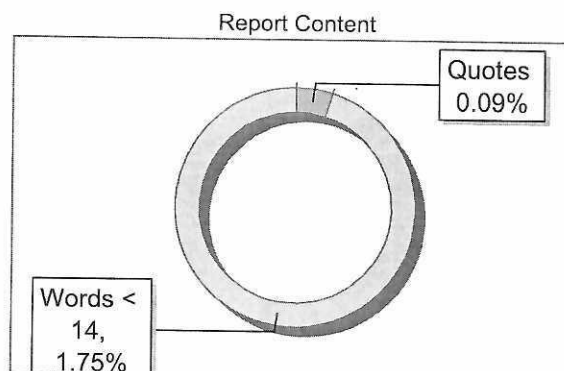
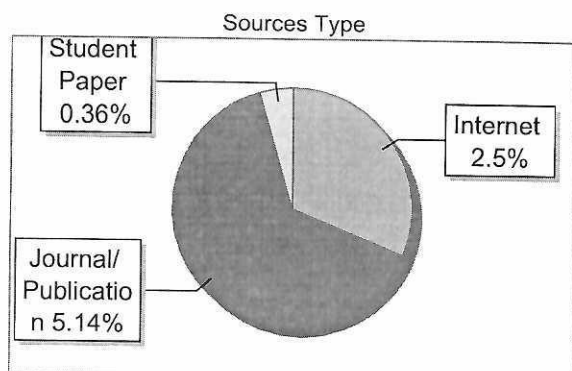
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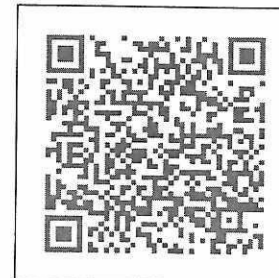
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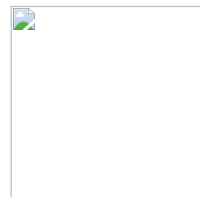
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ABSTRACT

This research entitled “Mathematical Study of the Transfer of Physical Properties in Newtonian Fluid Flow by Numerical Methods” presents a comprehensive and systematic investigation of fluid flow, heat transfer, and mass transfer phenomena through advanced mathematical modeling and numerical simulation techniques.

The study is motivated by the increasing demand for accurate prediction and optimization of transport processes in modern engineering systems such as thermal power plants, aerospace vehicles, chemical reactors, microelectronic cooling devices, and environmental systems. Fluid flow problems are governed by nonlinear partial differential equations derived from conservation laws. These include the continuity equation, momentum equation, energy equation, and species concentration equation. In realistic engineering situations, these equations are strongly coupled and subjected to complex boundary conditions, making analytical solutions extremely difficult or impossible. Therefore, this research emphasizes the role of numerical methods as a powerful and reliable tool for solving such complex problems. The study begins with the formulation of the mathematical model based on boundary layer theory. Boundary layer approximation significantly simplifies the governing equations while preserving the essential physics of viscous flows near solid surfaces. This concept is fundamental in analyzing aerodynamic drag, heat transfer rates, and flow behavior over stretching surfaces and porous media.

Similarity transformations are applied to convert the governing partial differential equations into a system of nonlinear ordinary differential equations, which are more suitable for numerical treatment. A major component of this research is the study of magnetohydrodynamic flow. The interaction of magnetic fields with electrically conducting fluids introduces the Lorentz force, which alters the velocity and temperature distribution within the fluid. The inclusion of Hall current effects further enhances the model by accounting for situations involving strong magnetic fields and low-density ionized fluids. These effects are particularly relevant in plasma physics, nuclear reactors, and electromagnetic pumping systems. Heat transfer analysis forms another important aspect of this research.

The study investigates conduction, convection, and radiation mechanisms in detail. Convective heat transfer is examined under natural, forced, and mixed convection conditions. Thermal radiation is incorporated using appropriate approximations, making the model suitable for high-temperature applications. Additional effects such as viscous dissipation and Joule heating are also included to

improve the accuracy of the model. In addition to heat transfer, the study explores mass transfer or species transport phenomena. Both diffusive and convective transport mechanisms are considered, along with cross-diffusion effects such as the Soret and Dufour effects. These phenomena are important in chemical processing, environmental engineering, and biological systems where temperature and concentration gradients coexist.

The research also focuses on nanofluids and hybrid nanofluids, which are advanced heat transfer fluids with enhanced thermal properties. The Modified Buongiorno Model is employed to account for nanoparticle dynamics such as Brownian motion and thermophoresis. These mechanisms significantly improve heat transfer performance and are widely used in modern cooling technologies. Fluid flow through porous media is another key area of investigation. The study considers both Darcy and non-Darcy flow models to analyze the effects of permeability and porosity on fluid behavior. Applications of porous media flow include groundwater hydrology, geothermal systems, and filtration processes. Chemical reaction effects and activation energy are incorporated into the model to simulate reactive flows. Entropy generation analysis is also performed to evaluate the thermodynamic efficiency of the system and identify sources of irreversibility. This analysis is useful in optimizing energy systems and improving performance. The numerical methodology adopted in this research includes advanced techniques such as the shooting method and finite difference methods. These methods are implemented using computational tools to obtain accurate numerical solutions of the transformed equations.

The results are validated by comparison with existing studies available in the literature. The results are presented in graphical and tabular form to illustrate the effects of various physical parameters on velocity, temperature, and concentration profiles. It is observed that the magnetic parameter reduces fluid velocity due to the Lorentz force, while increasing temperature profiles. Nanoparticles enhance heat transfer rates significantly, and slip conditions reduce surface friction. Porous media and radiation effects also play an important role in modifying flow behavior. The study concludes that numerical methods provide an efficient and reliable approach for solving complex fluid flow problems. The findings contribute to the advancement of knowledge in fluid mechanics and heat transfer and have practical applications in engineering and industrial processes. Future research directions include extending the analysis to non-Newtonian fluids, three-dimensional flow models, and the use of more advanced computational techniques for improved accuracy and efficiency.

INTRODUCTION

1.1 OVERVIEW OF THE STUDY

From hypersonic aircraft to common home appliances, border layer flow is a key component in many contemporary technological products. A thorough understanding of fluid flow concepts is critical. Building systems to transport water & natural gas makes use of fluid flow concepts. This is also true for the conduit network of the refrigeration & cooling system. Refrigerators use a compressor to expel the refrigerant, which then flows via internal tubes.

Fluid flow in automobiles also has a wide variety of uses. Flow visualisation, or the visual evaluation of flow field features, may substantially aid border layer issue analysis, in contrast to the complex mathematical computations required for quantitative fluid flow analysis. When it comes to numerical answers & practical studies, flow visualisation is a lifesaver. There are a variety of physical & statistical ways to depict various flow patterns graphically. Research into fluid movement takes into account the fact that, down to the millisecond, circumstances within a fluid body may change in both space & time. When the amplitude & direction of the velocity at any given place in the fluid are constant, we say that the flow is uniform. If the velocities vary at the given moment, we say that its motion is non-uniform. The region immediately next to the border layer will see changes in velocity as a fluid really moves close to the barrier in real life. On the other hand, when the dimensions & form of a fluid stream remain consistent, we say that its cross-section is uniform.

Heat transmission, pressure reduction, & drag force is all impacted by fluid motion. You can solve the energy equation quantitatively as velocity is a part of it. The heat flux via the flow field may then be measured. Estimating the flow's velocity distribution is also required for Heat exchange computations via convection. Advanced fields including biology, IT, security, & renewable energy rely on heat transmission, as do nuclear power plants, turbines, the paper industry, & refrigeration pipe systems.

The primary objective of this research is to achieve major progress in the field of Heat exchange, especially with regard to applications in the industrial sector. The term "Heat exchange" refers to the way in which heat is transferred from a hot object to a cold object. The process of Heat exchange takes place anytime there is a change in temp in relation to two bodies, regardless of how physically near or far away they are relative to one another.

At the moment when the internal & exterior temp of the body is same, it takes place. In accordance with the second law of thermodynamics, heat will always travel from items that are hotter to those that are cooler. One strategy that has emerged as a result of the substantial study that has been conducted into the development of methods for Heat exchange analysis is the use of cutting-edge technology to strengthen the characteristics of working fluids that are already in use & to create whole new types of working fluids.

Additionally, convective Heat exchange has a substantial impact on the surface temp field as well as the surface velocity of a fluid flow. There has been a continuous incorporation of the border layer concept into the model in recent times. Convective Heat exchange may be examined & made easier by focussing on the velocity & temp fields close to the solid surface. When dealing with an endless stream of fluid, it might be challenging to pick an appropriate method for evaluating its flow. Determining which part of this stream will serve as the investigated element or system & which parts will be considered as its environment & affects is essential.

In recent years, nanofluids have been introduced as a potential answer to the problem of Heat exchange in fluid flow issues. When it comes to the research of temp & velocity fields, these fluids are beneficial owing to the thermophysical features that they possess. Brownian motion, particle diameter, & the creation of nanolayers are all factors that have an effect on the enhancement of heat transmission (Nanofluids), in addition to thermal conductivity, particle size, & shape. Nevertheless, further research is still required in order to make a prediction about the maximum character that nanofluids are capable of possessing in comparison to the basic fluid. Nanofluids are a unique kind of fluid that include particles on the nanometre scale. These fluids exhibit

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extraordinary increases in Heat exchange qualities even at extremely low concentrations of Nano-objects. Nanofluids are not only a type of fluid.

As a result, it has the potential for usage in nuclear reactors, electronics, food, transportation, biology, & many other commercial applications. Some have even proposed using nanofluid as a smart fluid to regulate or enhance Heat exchange. A few examples of the many technological uses for nanofluids include in computer cooling systems, power plant cooling systems, & the medical field. The magnetic nanofluid is an uncommon material with magnetic & liquid properties. It is possible to alter the fluids' physical properties by applying a MF. The use of these organisms as models has also been very beneficial to fundamental scientific inquiry. Magnetic nanofluids have proven useful in a variety of scientific applications due to the ease with which they may be controlled by introducing an external MF to the flow of the fluid. Brownian diffusion & thermophoresis are the two processes that drive nanofluid slip. It is common knowledge that nanofluids are a unique category of Heat exchange fluids that result from the combination of Nano-objects with a base fluid. It is possible to increase the Heat exchange rate by adding certain components to the base fluid. There are several flow issues that come along with using this technology. Modern cooling rate needs aren't satisfied by the fluid's thermal conductivity, even when it comes to little Heat exchange.

The convectional Heat exchange capacities & thermal conductivity of the basic liquids may be enhanced using nanofluids, as previously shown. A possible explanation is that the Nano-objects move in a Brownian motion within the base fluids. Reasons for the varying enhancements in thermal conductivity of the nanofluid particles. The in response to potential energy supply issues, the international nuclear power industry is proposing a number of nuclear reactor designs that are both more cost-effective & safer. What the present border value flow results are Problems with nanofluids show that Brownian variables affected Heat exchange. Velocity, thermal conductivity, & fluid particle diameter. There needs to be further investigation. important for estimating the Nanofluid's maximum potential compared to the base substance.

The fluid was considered above a stretched sheet. In this work, we have treated the Carreaunanofluid as a generalised Newtonian fluid & examined the behaviour of

nanofluid flow patterns of significant parameters on different geometries, taking into consideration all aspects of the nanofluid. Carreau nanofluids are recognised as generalised Newtonian fluids when the shear rate is small. Carreau fluids exhibit non-Newtonian behaviour at high shear rates, defined as shear rates below 1. An issue with laminar nanofluid flow, which develops when a vertical surface is exposed under varying flow conditions, has been quantitatively examined in this work. By using computer-based simulation, systems like Heat exchange, fluid flow, & associated phenomena like turbulence & boundary layers may be examined. The compressible Navier-Stokes equations govern all of these processes. An analytical solution is required since many of the crucial elements of these equivalencies are significantly present in a non-linear way. Finding the right answer to a system of PDE is also possible with the help of numerical techniques.

1.2 MAGNETO HYDRO DYNAMICS (MHD)

A MF's effect on fluid flow may be seen in many contexts. The use of a MF facilitates the processing of metals in liquid form. A thermonuclear fusion reactor uses a MF to separate the heated plasma from the reactor's walls. Fluids that conduct electricity, such as electrolytes, plasma, liquid metals, & many more, are the focus of MHD. The main idea behind MHD theory is that when a fluid is moving through a MF, it creates an electric current. This current, in turn, causes the Lorentz force to act, altering the fluid's path. In 1832, the English physicist Michael Faraday conducted the first ever documented investigation on MHD. The electric current caused by the Thames River's velocity in Earth's MF was something Faraday aimed to quantify. Unfortunately, the experiment did not succeed since galvanometers of that era had a very limited capacity for detection. After Swedish electrical engineer Hannes Alfvén discovered Alfvén waves in 1942, MHD became well-known among scholars. A kind of wave-like oscillation that takes place in MF lines in a fluid that is highly electrically conductive is referred to as Alfvén waves, as stated by Davidson (2016) & Hosking & Dewar (2016). In 1970, Alfvén was honoured with the Nobel Prize in Physics in recognition of the significant & invaluable contributions he had made to the field of MHD. Magnetic Hall Effect (MHD) is used in a wide variety of industries, including but not limited to the following: MHD power generators, magnetic drug targeting, magnetic

endoscopy, nuclear reactor maintenance, study on solar flares & sunspots, geomagnetic storm analysis, crude oil refineries, & the design of MHD pumps & brakes.

1.2.1 Governing Equations of MHD Flow

A combination of the simplified version of the Maxwell electromagnetic laws & the conservation laws of fluid motion is used to develop the MHD flow equations (Davidson, 2016). Since the fluid in MHD flow is electrically neutral, the charge density is disregarded. The displacement current in a conducting fluid is negligible since the fluid's velocity is negligible when compared to the speed of light. At low magnetic Reynolds numbers, the generated MF has little to no impact (Cramer & Pai, 1973). The simplified version of the Maxwell equations, based on these assumptions, is presented in Table 1.1.

Table 1.1: Reduced form of the Maxwell equations

Name	Equation
Ampere's law	$\nabla \times \vec{B} = \mu_m \vec{J}$
Solenoidal property of magnetic field	$\nabla \cdot \vec{B} = 0$
Faraday's law	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Law of charge conservation	$\nabla \cdot \vec{J} = 0$
Ohm's law	$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B})$
Lorentz force	$\vec{F}_L = \vec{J} \times \vec{B}$

In where μ_m denotes the magnetic permeability. Therefore, the following equations regulate the MHD flow of a vector-form incompressible fluid:

Eq. of continuation

$$\nabla \cdot \vec{V} = 0, \quad (1.1)$$

Equation of momentum

$$\rho \left(\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right) = -\nabla p + \mu(\nabla^2 \vec{V}) + \vec{F}_L + \rho \vec{F}_b, \quad (1.2)$$

Power formula

$$\rho C_p \left(\frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla) T \right) = \kappa(\nabla^2 T) - \nabla \cdot \vec{q}_r + \phi + \frac{\vec{J} \cdot \vec{J}}{\sigma} + S_T, \quad (1.3)$$

Species concentration equation

$$\frac{\partial C}{\partial t} + (\vec{V} \cdot \nabla) C = D_m(\nabla^2 C) + S_C, \quad (1.4)$$

Where $\vec{V}$, T , C , p , & t are the velocity, temp, species concentration, pressure, & time, respectively. Here, $\rho \vec{F}_b$, $\vec{q}_r$, ϕ , $(\vec{J} \cdot \vec{J})/\sigma$, S_T , & S_C denote the external body force per unit volume, radiative heat flux, viscous dissipation term, Joulian dissipation term, temp source term, & concentration source term, respectively.

1.3 HALL CURRENT

American physicist Edwin Herbert Hall, while completing his doctorate at the esteemed Johns Hopkins University in Baltimore, Maryland, USA, in 1879, discovered the Hall effect. Within a transverse MF, charges on a current-carrying conductor are redirected in an orthogonal direction to the electric current & the MF. Therefore, the Hall current flows because a secondary potential difference is produced. When dealing with weak ionised gas densities or strong MFs, the Hall effect becomes practically relevant. EH is the induced electric field as a result of the Hall effect, which is related to the MF & the current density.

$$\vec{E}_H = R_H(\vec{J} \times \vec{B}), \quad (1.5)$$

Where $R_H = -1/(en_e)$ Where R_H is for the Hall coefficient, e for the charge of an electron, & n_e for the density of electrons. Taking into account the Hall effect in MHD flow, the generalised Ohm's law was defined by Cowling (1957) as:

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}) - \frac{\sigma}{en_e}(\vec{J} \times \vec{B}) + \frac{\sigma \nabla P_e}{en_e}, \quad (1.6)$$

In which the thermoelectric pressure is denoted as P_e . Several technical & industrial processes rely on the MHD flow with Hall current. These include magnetometers, electric transformers, plasma jets, ionised gas propagation, Hall accelerators, electric transformers, fibre & granular insulation, & magnetic resonance imaging (MRI).

1.4 NEWTONIAN FLUID

An ideal Newtonian fluid would exhibit a stress that grows in direct proportion to the strain rate, as stated in Newton's law of viscosity. Air, water, fuel, glycerol, mercury, and other similar substances are examples of Newtonian fluids. The Cauchy stress tensor (T) has the following form for an incompressible Newtonian fluid:

$$T = -pI + S, \quad (1.7)$$

In this case, the deviatoric stress tensor S is defined as follows, where I is the identity tensor:

$$S = \mu \left(\nabla \vec{V} + (\nabla \vec{V})^T \right), \quad (1.8)$$

Where superscript T represents transpose.

1.5 NONLINEAR FLUID MODEL

The stress-rate relationship in Nonlinear fluid models is nonlinear & depends on the strain rate. The Newtonian model does not apply to a wide variety of fluids used in industry & biology, including blood, gypsum paste, adhesives, lubricants, gels, paints, polymers, & synovial fluid. At a constant pressure & temp, the apparent viscosity of a Nonlinear fluid model changes depending on the flow shape, strain rate, & deformation

history (Chhabra & Richardson, 2008). Various Nonlinear fluid model types & Newtonian fluids are shown in Fig. 1.1.

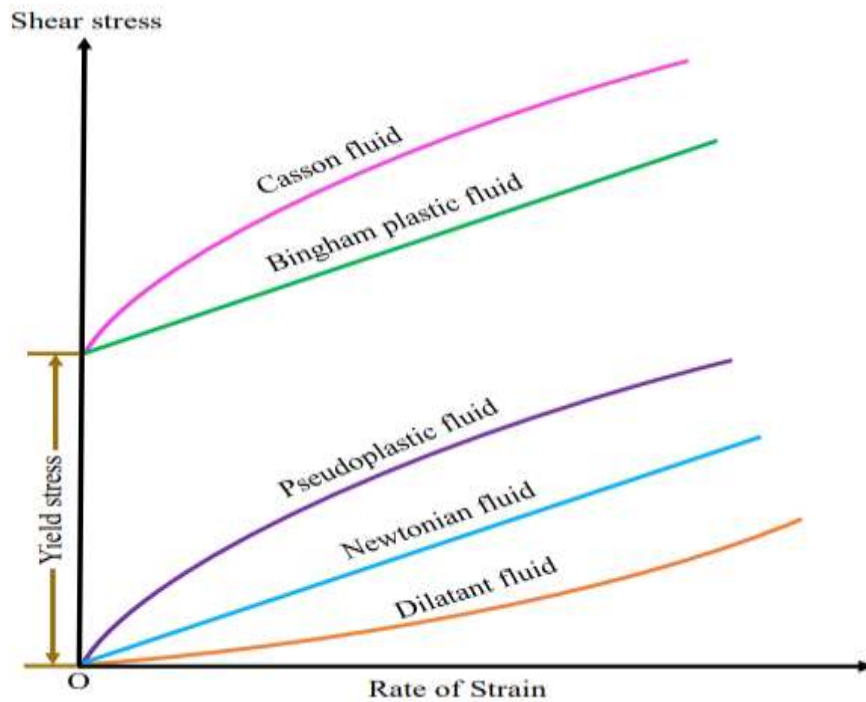


Figure 1.1: Newtonian and non-Newtonian fluids Rheogram

The apparent viscosity of pseudoplastic fluids, sometimes called shear-thinning fluids, decreases as the strain rate increases. Some fluids have shear-thinning properties; they include hair wash, paint, and ketchup. Shear thickening and dilatant fluids exhibit the opposite trend when it comes to the apparent viscosity of the fluid of interest. Shear-thickening fluids include mixes of cornflour and water, wet sand, and china clay suspensions, among other examples. In order for a Bingham plastic fluid to exhibit the characteristics of a Newtonian fluid, the shear stress that is applied to the fluid must be larger than a certain threshold value that is referred to as the yield stress. In the course of this thesis, the circulation of fluids models known as the Casson fluid, the second-grade fluid, and the Walters B' fluid are used in order to examine the rheological features of non-Newtonian fluids respectively.

1.5.1 Casson Fluid

Casson fluid is defined by yield stress & is shear-thinning. Lots of fluids exhibit the properties of the Casson model. Tomato sauce, custard, melted chocolate, jelly,

yoghurt, human blood, foams, & cosmetics are only a few examples. Many fields use the Casson fluid model, including biotechnology, food processing, metallurgy, geophysical fluid dynamics, & blood oxygenators. In 1959, the constitutive equations for the Casson fluid were presented in (Casson 1959). Based on Sahoo & N&keolyar's work in 2021, the Casson fluid's stress tensor component (τ_{ij}) may be expressed as:

$$\tau_{ij} = \begin{cases} 2 \left(\mu_B + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\ 2 \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \pi < \pi_c \end{cases} \quad (1.9)$$

Where e_{ij} , μ_B , p_y , & π_c are the strain-rate component, plastic viscosity of Casson fluid, yield stress, & critical value of the product $\pi = e_{ij}e_{ij}$, respectively. For $\pi < \pi_c$, the stress tensor of the Casson fluid is:

$$\tau_{ij} = 2\mu_B \left(1 + \frac{1}{\beta_C} \right) e_{ij}, \quad (1.10)$$

Where $\beta_C = (\mu_B \sqrt{2\pi_c})/p_y$ is the Casson parameter. The Casson fluid displays Newtonian behavior when $\beta_C \rightarrow \infty$.

1.5.2 Viscoelastic Fluid

A subset of Nonlinear fluid models, viscoelastic fluids may be defined as having both viscous & elastic characteristics. Withdrawal of shear stress in viscoelastic fluid flow causes a partial reversal of strain. The memory characteristic is present in viscoelastic fluids, meaning that the shear stress at any given moment is affected by the deformation gradient's history. Soup bowl effect is a prime example. When you stir the soup in the bowl, the liquid will circulate in the same direction that you stir it. After we stop stirring, the soup will circulate backwards for a little while. Soup, DNA, paint, motor oil, polymer melts, synovial fluid, & many more substances exhibit viscoelastic behaviour. Various aspects of industrial & food processing processes use the MHD movement of viscoelastic fluids. Emulsions of crude oil, xanthan gum, mayonnaise, polymers, carbon super fibre synthesis, & capillary transport dynamics are a few examples.

1. Second-Grade Fluid

In 1960, Coleman & Noll worked out the constitutive equation for a fluid of second grade. According to (Priyadarsan&P&a, 2020), the following expression for T characterises the second-grade fluid:

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \quad (1.11)$$

Where α_1, α_2 represent the normal stress moduli, & $\mathbf{A}_1, \mathbf{A}_2$ denote the leading Rivlin-Ericksen tensors given as:

$$\begin{aligned} \mathbf{A}_1 &= \nabla \vec{V} + (\nabla \vec{V})^T, \\ \mathbf{A}_2 &= \frac{D\mathbf{A}_1}{Dt} + \mathbf{A}_1 \cdot (\nabla \vec{V}) + (\nabla \vec{V})^T \cdot \mathbf{A}_1, \end{aligned} \quad (1.12)$$

Where $D/Dt \equiv \partial/\partial t + \vec{V} \cdot \nabla$ is the material derivative. Dunn & Fosdick (1974) have demonstrated that for thermodynamic consistency, the intrinsic constants of the second-grade fluid must satisfy the Clausius-Duhem inequality:

$$\mu \geq 0, \alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0. \quad (1.13)$$

2. Walters B' Fluid

In 1964, Beard & Walters put out the constitutive relation of Walters B' fluid. The equation for T of the Walters B' fluid is derived from works by Mishra & colleagues. (2018) & Hayat & colleagues. (2004):

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 - k_0 \frac{\delta \mathbf{A}_1}{\delta t}, \quad (1.14)$$

Where k_0 is the coefficient of viscoelasticity, & $\delta/\delta t$ is the Oldroyd upper convected derivative of a tensor given as:

$$\frac{\delta \mathbf{A}_1}{\delta t} = \frac{\partial \mathbf{A}_1}{\partial t} + (\vec{V} \cdot \nabla) \mathbf{A}_1 - (\nabla \vec{V}) \cdot \mathbf{A}_1 - \mathbf{A}_1 \cdot (\nabla \vec{V})^T. \quad (1.15)$$

1.6 HEAT EXCHANGE

In situations when there is a disparity in tempIn relation to two different things,this is called Heat exchange. The three main processes that cause this heat exchange are radiation, convection, & conduction, as shown in Figure 1.2. Intermolecular interactions allow for the passage of heat from a substance's more energetic molecules to its less energetic ones; this process is known as conduction. When two objects with different temp come into physical touch with one another, heat is transferred via conduction. When we touch a hot plate of food, for instance, our h&s become heated. There are three distinct forms of convection, which are defined by the direction & strength of the forces acting on the fluid, & which transfer heat by means of the fluid's bulk motion. The three distinct forms of convective heat transport are shown in Figure 1.3.

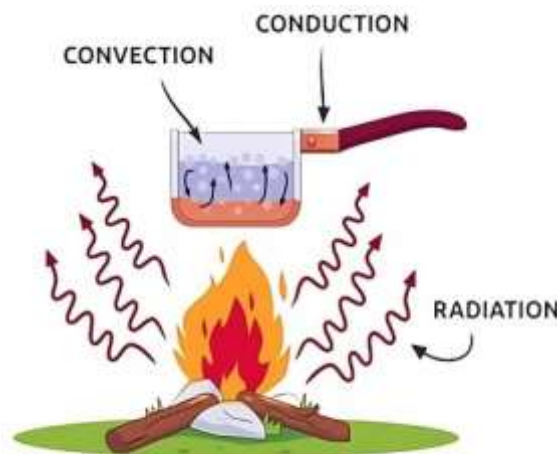


Figure 1.2: Modes of Heat exchange.

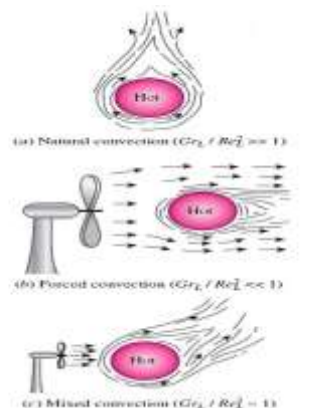


Figure 1.3: Modes of convective Heat exchange.

1. **Natural convection:** The process of natural convection begins with a shift in the fluid's density due to a change in temp, which then produces a buoyancy force. An instance of natural convection would be the cooling of a container holding water that has been heated. Thunderstorms, ocean currents, & sea breezes all originate from natural convection. Other names for natural convection include free convection & gravity-driven convection, which describe how the process occurs inside a gravitational environment. Natural convection cooling systems are the most cost-effective, energy-efficient, & least noise-polluting option. These include electric heaters, refrigeration coils, power transmission lines, hot air balloons, & solar collectors.
2. **Forced convection:** Forced convection occurs when a fan, pump, or suction device is used to propel the fluid's motion against its natural tendency. Forced convective fluid flow is used in many different applications, including air conditioning, ceiling fans, steam turbines, geysers, & the human circulatory system.

Mixed convection: When the buoyancy force is about equal to the external driving force, a condition known as mixed convection develops, in which natural & forced convection occur simultaneously. One example is the formation of a secondary natural convection current due to changes in fluid density brought about by low-velocity airflow over a hot vertical surface. Many situations call for mixed convective flow, including nuclear reactor emergency cooling, heat exchangers operating in environment with low velocity, as well as solar central receivers that are subjected to wind currents.

When electromagnetic waves are used to transmit heat, this process is called radiation. Radiative heat transmission, in contrast to convection & conduction, does not need a physical medium to take place. For example, light from the sun travels through space & time before reaching Earth. Radiation from a microwave oven is used to cook or heat food.

The radiative heat flux $(\vec{q}_r = (0, q_r, 0))$ two methods that represent this: the Rossel & approximation & the Engl & Emery approximation. For optically thick fluids, the

Rossel& approximation is employed, while for optically thin, grey, heat-absorbing or heat-emitting fluids in the absence of thermal radiation scattering, the Engl&& Emery approximation is utilised.

Rossel& approximation

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}, \quad (1.16)$$

Engl&& Emery approximation

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma^*(T_\infty^4 - T^4), \quad (1.17)$$

Where k^* is the Rosseland mean absorption coefficient and a^* symbolizes the absorption coefficient. Eqs. (1.16) and (1.17) are linearized by assuming $T - T_\infty$ to be sufficiently small and the truncated Taylor's series is $T^4 = 4T_\infty^3 T - 3T_\infty^4$. Eqs. (1.16) and (1.17) simplify as:

$$q_r = -\frac{16\sigma^*T_\infty^3}{3k^*} \frac{\partial T}{\partial y}, \quad (1.18)$$

$$\frac{\partial q_r}{\partial y} = -16a^*\sigma^*T_\infty^3(T_\infty - T). \quad (1.19)$$

Heat exchange in fluid flow problems is a hot topic among researchers because of all the cool things it can be used for: solar collectors, automobile radiators, cooling electronic circuit boards, rocket propulsion systems, beverage production, metallurgical processes, & precision glass moulding.

1.7 TRANSPORT OF SPECIES

Transport of species describes the movement of stuff from a densely populated area to an open one. When an alien species is present in a fluid, it creates a concentration gradient that makes Transport of species easier. One example is the colour shift that occurs when a little quantity of potassium permanganate is added to water, turning it purple. The scent of perfume wafts around the room when the wearer applies it.

Another example of Transport of species is the process of dissolving sugar in lemonade, which requires a teaspoon. Absorption of nutrients into the circulation, blood oxygenation, ion movement via osmosis, water purification, distillation, & many other biological & chemical processes rely on Transport of species. Both diffusion & convection play a role in the movement of mass through fluids. The species shown in Figure. 1.4 undergo diffusive Transport of species as a result of molecular mobility.

The transmission of physical attributes such as momentum, heat, & mass in Newtonian fluid flow involves substantial mathematical & computational hurdles. The Navier-Stokes equations in particular are notoriously difficult to solve analytically, especially when dealing with complex border conditions & geometries, despite the fact that Newtonian fluids are simplified by the assumption of constant viscosity. When applied to real-world scenarios, traditional analytical approaches fail to provide adequate solutions. Consequently, trustworthy numerical approaches must be used & developed to adequately manage these intricacies & provide precise forecasts of fluid behaviour & property transfer.

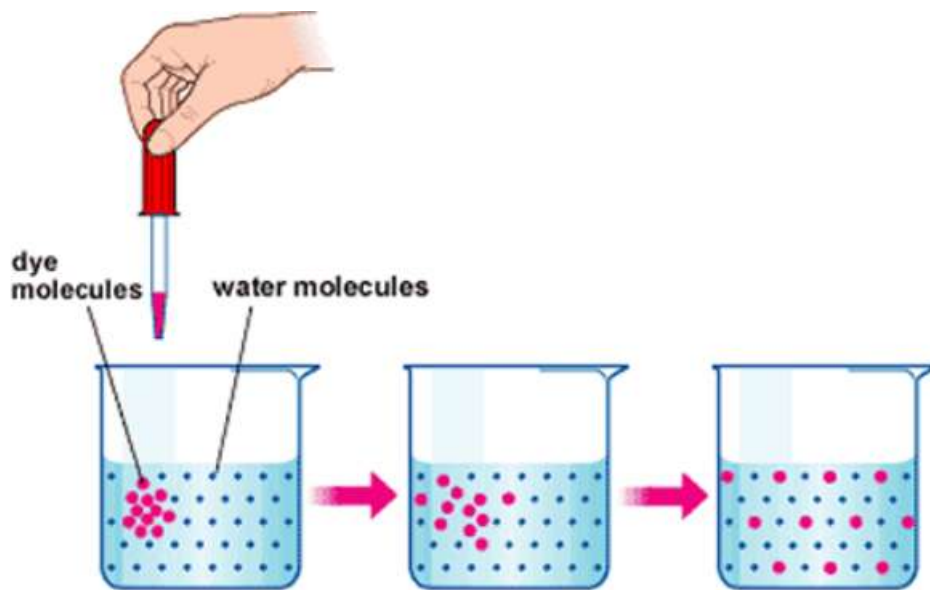


Figure 1.4: Propagation of dye molecules by diffusion

Conversely, convective Transport of species describes the situation where the species spreads together with the fluid's bulk flow. The glucose produced by plants during photosynthesis is transported to different sections of the plant by the water absorbed by the soil. Because of its relevance in many fields, including environmental engineering,

electrical engineering, metallurgy, refrigeration, pharmaceutical drug development, chemical constituent separation in distillation apparatus, HVAC system design, & automobile engineering, the study of MHD flow with heat & Transport of species has captivated many researchers.

1.8 CROSS-DIFFUSION EFFECTS

It is common for a concentration gradient to cause mass diffusion & a temp gradient to cause heat diffusion. Mass diffusion occurs as a result of temp gradients & vice versa when there are high concentration & temp gradients (Kafoussias & Williams, 1995). The mass flow that occurs as a consequence of a temp gradient is referred to as the Soret effect, while the heat flux that occurs as a consequence of a concentration gradient is referred to as the energy flux.

Swiss physicist Charles Soret discovered the Soret effect in 1879. Two tubes containing sodium chloride (NaCl) & potassium nitrate (KNO₃) solutions were placed in front of him for experimental observation. The tubes were either straight or U-shaped. Both tubes had one end in a bath of icy water & the other in a pot of boiling water. The experiment proved the existence of the Soret effect by showing that the solute settled to the bottom of the tubes as they were chilled. Species migrate from warm to cold areas as a result of the positive Soret effect, & vice versa as a result of the negative Soret effect (Platten, 2006). A favourable Soret effect is more often seen in heavier species, whereas a negative Soret impact is more commonly seen in lighter species. In 1873, the Dufour effect was discovered by a Swiss scientist called L. Dufour. As an example, the Dufour effect would cause a temp increase of several degrees in a combination of hydrogen & nitrogen, which are held at various concentrations. Rowley & Horne (1980) conducted experimental studies on the Dufour effect in a combination of cyclohexane & carbon tetrachloride. Gaseous mixes of intermediate molecular mass (Nitrogen & air) exhibit the Dufour effect, according to Eckert & Drake (1972), but combinations of gases of little molecular mass (Hydrogen & helium) exhibit the Soret effect. Isotope separation, fuel cell design, nuclear reactor engineering, petrology, & gas impurity removal is just a few of the many engineering fields that make use of the Soret-Dufour phenomena, which represent a cross-diffusive coupled heat & Transport of species in fluid flow.

1.9 CHEMICAL INTERACTION & ACTIVATION BENERGY

In a Chemical interaction, bonds in the reactant species are broken, allowing bonds in the product species to be formed. The rate of the Chemical interaction is determined by the concentration of the species that are responsible for the reaction. We refer to a reaction as being of first order when the concentration of the reactant species is directly proportional to the first power of the rate constant used in the reaction.

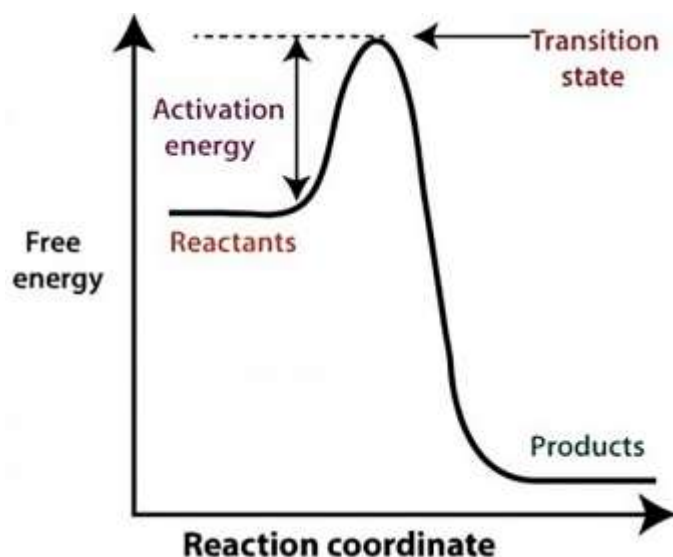


Figure 1.5: Activation energy.

There is a tight relationship in relation to the activation energy & the reaction rate in chemistry. Chemical interactions may only take place when the reactants reach a certain threshold energy, according to Swedish scientist Svante Arrhenius's 1889 discovery. It was Arrhenius who first used the word "activation energy" to describe the bare minimum of energy that molecules of the reactants needed to start a Chemical interaction. In order for the reactants to collide & start forming product species, they must overcome this energy gap, as depicted in Fig. 1.5.

The rate constant, activation energy, & temp are all variables in the Arrhenius equation, which, as stated by Tencer& colleagues. (2004), is:

$$k_c = A \exp \left(-\frac{E_a}{k_B T} \right), \quad (1.20)$$

Where A is the Arrhenius pre-exponential factor, E_a is the activation energy, and $k_B = 8.61 \times 10^{-5}$ eV/K is the Boltzmann constant. Numerous fields, including chemical engineering, geothermal engineering, food processing, oil emulsions, polymer manufacture, and ceramic or glassware production, benefit from studying MHD flow in relation to chemical reaction and activation energy.

1.10 POROUS MEDIUM

The solid matrix of a porous material has interconnected spaces called pores. As shown in Figure 1.6(a), the fluid is able to move across the medium thanks to the web-like network formed by the linked pores. Porous media include things like s&, wood, limestone, furnace refractory material, sponges, fabric, fibreglass, ceramics, & even human lungs. As shown in Figure 1.6(b), pumice is a very porous igneous rock that forms when lava is quickly cooled & depressurised after being expelled during a volcanic eruption. To measure porosity, take the volume of the pores & divide it by the volume of the medium. The porosity value for naturally porous medium is less than or equal to 0.6 (Nield&Bejan, 2017).

Permeability (k_p) is the ease with which a fluid flows through the porous medium.

An empirical relation for modelling fluid flow through porous media was established in 1856 by the French engineer Henry Darcy; it is now known as Darcy's law. The law has the following form:

$$\nabla p = -\frac{\mu}{k_p} \vec{V}. \quad (1.21)$$

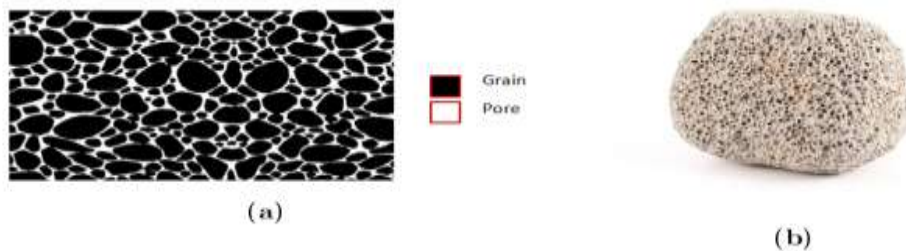


Figure 1.6: (a) Porous medium & (b) Pumice.

When the Reynolds number is small relative to the pore diameter, which indicates low-velocity flow, Eq. 1.21 holds. Most flows occurring below ground have Reynolds numbers that fall within the Darcy's range. On the other hand, in cases with steep gradients or with big solid particles in the flow, the fluid's trajectory takes a curved shape, which causes it to experience inertial acceleration. It follows that a non-Darcy equation must be used, taking into account both the inertia caused by the high velocity flow & the typical stress caused by the distortion velocity. As a result, a number of models that do not include Darcy have been developed. The Darcy-Forchheimer law, which was proposed by Forchheimer (1901) & is expressed as the quadratic form drag in Darcy's law (Eq. 1.21), is:

$$\nabla p = -\frac{\mu}{k_p} \vec{V} - \frac{\rho C_F}{\sqrt{k_p}} |\vec{V}| \vec{V}, \quad (1.22)$$

Where C_F is the non-dimensional form-drag coefficient. The study of convective flow in porous media is useful in many different contexts, including but not limited to: groundwater hydrology, magnetic-levitation casting, geothermal reservoirs, decontamination processes, alloy solidification, microporous heat sinks, blast furnaces, & catalytic converters.

1.11 NANOFUID & HYBRID NANOFUID

The use of effective coolants is essential for the upkeep of complex machinery and industrial equipment. Power generation, chemical manufacture, food processing, automobile engines, nuclear reactors, textile production, and microelectronics are just a few examples of industries that have a significant need for effective cooling techniques. A further important factor influencing the final product's quality is the transfer of heat rate between the solid surface and the fluid. In comparison to more conventional base fluids like water, motor oil, and ethylene glycol, the thermal conductivity of metals, oxides, and carbon nanotubes (CNTs) is often much greater. Nanotechnology has recently advanced to the point that it is feasible to add nanoparticles to the base fluid to improve its heat transmission capabilities. One kind of nanoparticle is colloiddally suspended in the base fluid of a nanofluid, making it an increased transfer of heat fluid. The ground-breaking study on the improvement of heat transmission in Cu H₂O nanofluid was conducted by Choi and Eastman (1995), who

also established the term "nanofluid". The single-phase and double-phase models are the main tools for studying nanofluid flow. According to the single-phase model, the base fluid and nanoparticles move at the same speed because of thermal equilibrium. According to Albojamal and Vafail (2017), the nanofluid's transfer of heat properties may be altered by focussing on the two phases of the model: the solid nanoparticle phase and the fluid phase. The rate of transfer of heat in the cavity flow of Cu - H₂O nanofluid was investigated by Tiwari and Das (2007) using the single phase model. As a function of the volume percentage of nanoparticles, the authors have given a detailed list of the thermo-physical characteristics of nanofluid. For the purpose of enhancing the thermophysical properties of nanofluids, researchers have developed a novel kind of fluid known as a hybrid nanofluid. This type of nanofluid has a mixture of Nano-objects of various types, which are combined with the base fluid. Devi & Devi (2016) have shown the correlations for the thermo-physical properties of hybrid nanofluids. These features include fluid properties. Due to their diverse applications in nano-drug administration, heat exchangers, refrigeration, solar collectors, supercomputers, autos, cryosurgery, cancer treatment, transformers, heat pumps, photoelectric equipment, textile industries, geothermal power extraction, & nuclear reactor maintenance, nanofluid & hybrid nanofluid flow research has recently attracted the interest of many researchers.

❖ The Modified Buongiorno Model

Buongiorno (2006) put out the Buongiorno model, which takes into account the Nano-objects' relative velocities (slip velocities) in relation to the base fluid. Buongiorno (2006) found seven different slip processes for Nano-objects; however, due to the lack of turbulent effects & the maximum size limit of 100 nm, he determined that thermophoresis & Brownian diffusion were the only two important ones. Brownian diffusion is the process by which Nano-objects move across a medium by constantly crashing against molecules of the base fluid. But when Nano-objects migrate along a temp gradient, this process is called thermophoresis. Yang & colleagues. (2013) were the first to use the Modified Buongiorno model (MBM), which examines the impact of nanoparticle volume fraction on the convective flow of nanofluids using the Buongiorno model equations. The study was carried out on nanofluids of Al₂O₃ – H₂O & TiO₂ – H₂O that were subjected to forced convective flow inside a concentric

annulus. The mixed convective nanofluid flow within a vertical annulus was studied by Malv&i& colleagues. (2014) using the MBM. When it comes to nanofluid convective transport, the governing equations, according to the MBM, are:

Continuity equation

$$\nabla \cdot \vec{V} = 0, \quad (1.23)$$

Momentum equation

$$\rho_{nf} \left(\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right) = -\nabla p + \nabla \cdot \mathbf{S}, \quad (1.24)$$

Energy equation

$$\begin{aligned} (\rho C_p)_{nf} \left(\frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla) T \right) = \nabla \cdot (\kappa_{nf} \nabla T) \\ + (\rho C_p)_s \left(D_B (\nabla C \cdot \nabla T) + \frac{D_T}{T} (\nabla T \cdot \nabla T) \right), \end{aligned} \quad (1.25)$$

Nanoparticle concentration equation

$$\frac{\partial C}{\partial t} + (\vec{V} \cdot \nabla) C = \nabla \cdot \left(D_B \nabla C + \frac{D_T}{T} \nabla T \right), \quad (1.26)$$

Where $\vec{V}$, $\mathbf{S} = \mu_{nf} (\nabla \vec{V} + (\nabla \vec{V})^T)$, T , and C denote the nanofluid velocity, deviatoric stress tensor for a Newtonian nanofluid, nanofluid temp, & nanoparticle concentration, respectively.

1.12 CATTANEO-CHRISTOV DOUBLE DIFUSION MODEL

The Fourier slaw expresses the heat flux ($\vec{q}$) as:

$$\vec{q} = -\kappa \nabla T, \quad (1.27)$$

Whereas the mass flux ($\vec{j}$) is represented by the Fick s law as:

$$\vec{j} = -D_m \nabla C. \quad (1.28)$$

The energy equation becomes parabolic when Fourier's heat flux & Fick's mass flow is both included, & the species concentration equation becomes parabolic as well. This means that an initial disturbance is instantly distributed over the whole domain. In other words, the "paradox of heat conduction" holds true, & the heat disturbance travels at an unlimited pace. Cattaneo (2019) offered the Maxwell-Cattaneo (MC) law as a solution to this unrealistic occurrence; this law incorporates the thermal relaxation time component into the Fourier's heat flow & is expressed as:

$$\vec{q} + \lambda_T \frac{\partial \vec{q}}{\partial t} = -\kappa \nabla T, \quad (1.29)$$

Where λ_T is the thermal relaxation time, which signifies the time lag for acquiring steady-state heat conduction in a fluid element exposed to a sudden temp gradient. Most materials have a thermal relaxation time that is negligible, measured in picoseconds at the most. As to Ch&rasedkharaiiah (1998), components such as s& (21s), glass ballotini (11s), ion exchange (54s), H⁻ acid (25s), & NaHCO₃ (29s) cannot be disregarded.

By using the MC rule, the energy equation takes on a hyperbolic form, suggesting that heat disturbances travel at a limited wave speed. Christov (2009) made an additional adjustment by substituting the Oldroyd upper convected derivative for the partial time derivative; this latter Expression generalises the MC rule to a frame-independent extent. The concurrent heat & mass diffusion represented via the Cattaneo-Christov theory is termed the Cattaneo-Christov double diffusion model, & the respective expressions for $\vec{q}$ and $\vec{j}$ are (Shankar & Naduvanamani, 2019):

$$\vec{q} + \lambda_T \left(\frac{\partial \vec{q}}{\partial t} + (\vec{V} \cdot \nabla) \vec{q} - \vec{q} \cdot (\nabla \vec{V}) + (\nabla \cdot \vec{V}) \vec{q} \right) = -\kappa \nabla T, \quad (1.30)$$

$$\vec{j} + \lambda_C \left(\frac{\partial \vec{j}}{\partial t} + (\vec{V} \cdot \nabla) \vec{j} - \vec{j} \cdot (\nabla \vec{V}) + (\nabla \cdot \vec{V}) \vec{j} \right) = -D_m \nabla C, \quad (1.31)$$

Where λ_C denotes the mass relaxation time. According to Rehman& colleagues. (2023), the Cattaneo-Christov double diffusion scheme accurately represents the heat

& mass transport processes caused by concentrated & high-temp gradients as well as by fields of changing concentration & temp.

1.13 ENTROPY GENERATION

Entropy is created during an irreversible process, according to the second rule of thermodynamics. Every time energy is transformed into usable work, some of that energy is lost, reducing the system's efficiency. Because of the inherent irreversibility's

in thermodynamic processes, entropy formation occurs in direct proportion to this energy deterioration. Entropy production in convective heat transmission was the subject of Bejan's seminal research (1979). The analysis takes into account the effects of conduction transfer of heat and viscous dissipation on the entropy development rate. When heat and mass transmission are occurring at the same time, Mourad and colleagues. (2006) examined how entropy is generated. So far as we can tell from Mourad and colleagues. (2006), the SG rate at the local level is:

$$\begin{aligned}
 S_G = & \frac{\kappa}{T^2} \left(\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right) \\
 & + \frac{\mu}{T} \left(2 \left(\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right) \\
 & + \frac{RD_m}{C} \left(\left(\frac{\partial C}{\partial x} \right)^2 + \left(\frac{\partial C}{\partial y} \right)^2 \right) \\
 & + \frac{RD_m}{T} \left(\left(\frac{\partial T}{\partial x} \right) \left(\frac{\partial C}{\partial x} \right) + \left(\frac{\partial T}{\partial y} \right) \left(\frac{\partial C}{\partial y} \right) \right),
 \end{aligned} \tag{1.32}$$

In where R st&s for the ideal gas constant. Here, Heat exchange by conduction (first term), viscous dissipation (second term), diffusive Transport of species due to concentration gradient alone (third term), & cross-diffusive Transport of species due to temp& concentration gradients (fourth term) all contribute to entropy generation. Improving the system's performance relies heavily on analysing entropy creation, which aids in reducing the elements that degrade energy. Numerous fields rely on entropy analysis, including biological systems, electronic cooling, geothermal energy generation, solar collectors, magnetic refrigeration, & solid-state physics.

1.14 BORDER LAYER THEORY

The border layer idea was first out by German physicist Ludwig Prandtl in 1904. When a thin layer of a low viscosity fluid passes over a stationary solid surface, there is no frictional loss, hence the flowing fluid will adhere to the solid. This immobile layer slows down subsequent fluid layers, and the process repeats itself after that. However, when one goes normal away from the bounding surface, the retarding effect on the fluid layers' decreases. The creation of a thin layer near the border surface, known as the border layer, causes the fluid velocity to rise from zero at the border surface to free stream velocity $(U_\infty(x))$.

The border layer experiences substantial forces due to rotation & viscosity. The outer layer, which goes beyond the border layer, is defined by inviscid & irrotational flow. A high Reynolds number is required for a fluid to form a border layer. With a small value of Re , the viscous forces would be noticeable across the whole flow domain, eliminating the need for a border layer. Similarities exist in the characterisation of the concentration & heat border layers when mass & Heat exchange take place at the border. Instead of using inviscid Euler equations to determine the viscous drag force, border layer theory may be used to define the no-slip condition at the solid border. In addition, by evaluating them in terms of magnitude, unnecessary components may be removed from the border layer flow Navier-Stokes equations, perhaps making them simpler to understand. Schlichting & Gersten provide an in-depth analysis of border layer theory & its practical uses (2017). For a two-dimensional Newtonian fluid with continuous incompressible flow & no body forces, the following are the Pr&tlborder layer equations, as stated by Bansal (1977):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1.33)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty(x) \frac{dU_\infty(x)}{dx} + \nu \frac{\partial^2 u}{\partial y^2}, \quad (1.34)$$

subject to:

$$\begin{cases} u = 0, v = 0 \text{ at } y = 0, \\ u \rightarrow U_\infty(x) \text{ as } y \rightarrow \infty. \end{cases} \quad (1.35)$$

A plethora of practical engineering problems have found a solution in border layer theory. These include, but are not limited to, minimising aerodynamic drag on cricket balls & aeroplane wings, determining hydrodynamic drag on submarines, ships & turbines, designing heat exchangers, cooling & drying paper sheets, processing polymers & countless more.

1.15 SIMILARITY TRANSFORMATION

In a two-dimensional border layer flow with velocity field $\vec{V} = (u(x, y), v(x, y))$, the non-dimensional velocity component $u/U_\infty(x)$ varies with only one variable $\eta = y/g(x)$, where $g(x)$ is proportional to the boundary layer thickness ($\delta(x)$), & η is called the similarity variable (Som, & colleagues., 2012; Bansal, 1977; Kundu & Cohen, 2004). This change, known as the similarity transformation, converts the dimensional system of PDEs into a non-dimensional system of ODEs that are conceptually identical. German physicist Heinrich Blasius first proposed the similarity transformation in 1908 as a solution to the border layer flow issue across a flat plate. A Newtonian fluid's continuous incompressible flow across a semi-infinite flat plate with a constant free stream velocity (U_∞) was something that Blasius took into consideration. The continuity equation (1.33) is identically satisfied by considering the dimensional stream function $\psi(x, y)$ as:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \quad (1.36)$$

The definition of the similarity transformation relations is based on Kundu & Cohen (2004):

$$\eta = \frac{y}{x} \sqrt{Re_x} = y \sqrt{\frac{U_\infty}{\nu x}}, f(\eta) = \frac{\psi}{\sqrt{\nu x U_\infty}}, \quad (1.37)$$

Where $f(\eta)$ denotes the non-dimensional stream Expression, & $Re_x = \frac{U_\infty x}{\nu}$ is the local Reynolds number.

1.16 SUCTION/INJECTION

To manage the border layer's development, suction/injection applications are critical. To keep the border layers from separating & to increase the lift coefficient, suction is an essential tool in aerodynamic engineering. The slowed-down molecules of the fluid are suctioned into the border layer flow, which reverses the negative pressure gradient at the bordering surface. Molecules in the fluid that are moving quickly work against those that are moving slowly to lessen the possibility of border layer separation. Also, the kinetic energy given off by the tangential injection of fluid into the border layer makes flow separation more difficult. The impact of suction on an airfoil's aerodynamic lift coefficient was studied by Shoejaefard & colleagues. (2005). The findings show that injection lowers the Wall shear coefficient, whereas suction increases the lift coefficient. Spacecraft re-entry, rocket engine propellants, ablative & heat sink cooling, & transpiration cooling the process of sucking or injecting fluid through porous surfaces have industrial & technical uses.

1.17 DIMENSIONAL ANALYSIS

Researching flow behaviour on a full-scale model is a laborious, costly, & complex process. For this reason, researchers often choose to test the prototype's flow behaviour on a smaller scale before making any firm predictions about the prototype's actual performance. The non-dimensional factors ensure that the model flow & the prototypical flow are identical (White, 2011; Cengel & Cimbala, 2014). It is common for fluid dynamics issues to have four main dimensions: mass, length, time, & temp. These dimensions are used to construct the secondary dimensions. The dynamic viscosity dimension, for instance, is given by $[M L^{-1} t^{-1}]$. An essential tool for non-dimensionalizing the governing equations is dimensional analysis. By combining the issue's dimensional parameters into a collection of non-dimensional parameters, we may simplify the problem & do fewer trials. Adopting the Buckingham Pi theorem, established by Buckingham (1914), yields the non-dimensional parameters. Dimensional analysis is useful for a number of reasons, including deciphering experimental results, tackling issues that are too complex for straightforward techniques to handle mathematically, & determining the relevance of a particular physical occurrence.

1.18 BOUSSINESQ APPROXIMATION

The buoyant force components in the equations that regulate natural convective flow are approximated using the Boussinesq method. According to Incropera & colleagues. (2006), the momentum equation is the only one that takes into account the fluctuation of fluid density in incompressible natural convective flows. The thermal expansion coefficient (β) & the mass expansion coefficient (β^*) are given by:

$$\beta = -\frac{1}{\rho_0} \left(\frac{\partial \rho(T, C)}{\partial T} \right)_p, \quad \beta^* = -\frac{1}{\rho_0} \left(\frac{\partial \rho(T, C)}{\partial C} \right)_p, \quad (1.38)$$

Where $\rho_0 = \rho(T_0, C_0)$ is the reference fluid density at the reference temperature (T_0) and concentration (C_0). Assuming that temperature and concentration differences are small, the truncated Taylor's expansion of $\rho(T, C)$ about (T_0, C_0) gives us:

$$\rho = \rho_0 (1 - \beta(T - T_0) - \beta^*(C - C_0)). \quad (1.39)$$

Thus, the buoyancy force per unit volume of the fluid via the Boussinesq approximation assumes the form:

$$-\rho g = -\rho_0 g + \rho_0 g \beta (T - T_0) + \rho_0 g \beta^* (C - C_0). \quad (1.40)$$

1.19 BORDER CONDITIONS

To solve the system of DE that regulates the flow, border conditions are established at the border of the flow domain. These conditions are employed to solve the system. In fluid dynamics, some of the border conditions that are often encountered include the Dirichlet, Neumann, & Robin's. At $y = 0$, the circumstances at the solid surface are given, whereas at $y \rightarrow \infty$, the ambient conditions are given.

1.19.1 No-Slip Condition

The majority of investigations have shown that as fluids move over solids, a thin film of fluid sticks to the surface. The no-slip condition which asserts that at all fluid solid interfaces, the relative velocities of the fluid & the solid surface are zero represents this quality. One way to put it is as:

$$u = U_w \text{ at } y = 0, \quad (1.41)$$

where U_w is the velocity of the solid surface.

1.19.2 Slip Condition

The slide condition should be used instead of the no-slip criterion in several physical contexts. As an example, when dealing with gas flows that are somewhat rarefied, the slip condition is applicable. According to Wang and colleagues. (2021), the slip condition is crucial for creating nanofluidic devices. According to Hatzikiriakos (2015), slip property is shown by a number of non-Newtonian fluids, including microgels, suspensions, elastomers, polymer melts, and foams. At each location along the border between the fluid and the solid, the slip condition presupposes a relative velocity (slip velocity) between the two. As an expression, the slip condition is:

$$u = U_w + K_1 \frac{\partial u}{\partial y} \text{ at } y = 0, \quad (1.42)$$

Where K_1 is the velocity slip factor.

1.19.3 Newtonian Heating

Newtonian heating is the process wherein heat is transferred from the surrounding surface to the fluid in direct proportion to the local surface temp. Conjugate Heat exchange across fins, the petroleum industry, heat exchangers, & solar collectors are all places where this phenomenon is common. Merkin (1994) was the first to explain the Newtonian heating condition, which is shown as:

$$\frac{\partial T}{\partial y} = -h_1 T \text{ at } y = 0, \quad (1.43)$$

Where h_1 represents the heat coefficient.

1.19.4 Double Stratification

Layers of fluid with varying densities are created by variations in fluid temp& concentration; this process is called twofold stratification. It is common to see double stratification in solar collectors, groundwater reservoirs, smog generation, plastic

extrusion, & atmospheric heat sinks. Per Srinivasacharya&Uppendar (2013), the relevant environmental constraints are:

$$T \rightarrow T_{\infty}(x), C \rightarrow C_{\infty}(x) \text{ as } y \rightarrow \infty, \quad (1.44)$$

Where $T_{\infty}(x) = T_0 + d(x/l)$ and $C_{\infty}(x) = C_0 + e(x/l)$, respectively. Here, T_0 , C_0 , d , and e denote the reference temperature, concentration, and dimensional constants, respectively.

1.20 RESEARCH OBJECTIVES

1. To obtain a mathematical model of the basic transfer of heat process for various nonlinear problems of nanofluid flows.
2. To calculate transfer of heat coefficients to find the numerical solution to these problems.
3. To identify the parameters & their characteristics that enhances the transfer of heat coefficient.
4. To find the parameters responsible for the reduction of skin friction.
5. To research the influences of various parameters on the flow field at border layer as well as transfer of heat characteristics of nanofluids.

1.21 IMPORTANCE

Understanding & accurately predicting the transfer of physical properties in Newtonian fluids is essential in various engineering & industrial processes such as pipeline design, heat exchangers, chemical reactors, & environmental flow systems. The use of advanced numerical methods can significantly enhance the precision of simulations, leading to better design, optimization, & operational efficiency. This research is important because it provides practical solutions to fluid dynamics problems that cannot be solved analytically, thereby contributing valuable insights to both academic research & industry applications.

1.22 SCOPE & LIMITATIONS

1. Scope of the study

- The objective of the project is to compute the Heat exchange coefficient & find the numerical solution to a fluid issue. However, this research can potentially be expanded in the future to include diverse types of fluids, as Nonlinear fluid models also have unique properties. It is possible to improve Heat exchange by utilizing the characteristics of these fluids. Different geometries, such as conical forms, spherical cylinders, & moving surfaces, may be studied in the future when it comes to fluid flow issues. Future extensions of the work will address additional characteristics such as the double diffusion phenomena, ohmic heating, viscous dissipation, & distinct chemical processes.
- Another fascinating area of border value problem-solving computational techniques is the numerical approach with Matlab. In the future, improving the numerical techniques may also be utilized to increase the accuracy of the system of equations' solution.
- The connection In relation to Bernoulli's equation & Euler's equation, which is produced when the equation of momentum is applied to a uniform, steady flow of inviscid material. Future research on flow analysis on pipes, ducts, & open channels can also make use of the steady flow energy equation. The industry has a bright future for the analysis of frictional or separation losses to available energy in the fluid network to the pumps or turbine.

2. Limitations of the study

- The study exclusively considers Newtonian fluids, where the viscosity is constant & independent of the shear rate.
- For computational feasibility, the study often employs idealized border& initial conditions.

- The simulations are restricted by available computational power & time. More refined grids or smaller time steps, which could potentially increase accuracy, are sometimes avoided due to high computational costs.
- If the study is limited to two-dimensional flow analysis, the results may not fully represent three-dimensional flow behavior, which is often present in practical engineering & physical systems.

1.23 STRUCTURE OF THE THESIS

Chapter 1 provides a thorough overview of the basic concepts of Heat exchange & fluid movement. The chapter starts with MHD investigation, including MHD flow governing equations, & then moves on to the Hall current effects. Newtonian & Nonlinear fluid models, including viscoelastic & Casson fluids, are also covered in terms of their properties & behaviour. We take a close look at the mass & heat transmission, as well as the relevance of the cross diffusion effects. Chemical interactions, activation energy, & fluid flow behaviour in porous media are also discussed in this chapter. As an example of current progress, the ideas of nanofluid & hybrid nanofluid flow are presented. The Cattaneo-Christov double diffusion model & the modified Buongiorno model are used as more examples of models. Dimensional analysis, similarity transformation, entropy generation, border layer theory, suction/injection, & the Boussinesq approximation are all treated in detail. Also included are a number of border conditions, such as no-slip, slip, Newtonian heating, & double stratification. Laplace transforms & MATLAB solvers were used for numerical solutions, & the chapter concludes by outlining the technique & literature study that were utilised.

In Chapter 2, the subject of nanofluid flow over stretched or extended surfaces is thoroughly reviewed in the literature. The chapter begins with a research background that outlines the main concepts & past accomplishments in the subject of nanofluid dynamics, laying the framework for the study of this topic. An in-depth examination of the stretching surfaces problem with Williamson nanofluid is conducted to bring attention to the unique rheological properties & flow behaviour of this Nonlinear fluid model type. In this enlarged study, the significance of micro-rotation & spin variables in the dynamics of micropolar nanofluid border layer flow over stretching surfaces is

further investigated. Also discussed are power-law nanofluid flow analyses across extended surfaces, which help to clarify how non-linear viscosity influences both flow & Heat exchange. Lastly, the chapter establishes the framework for the present investigation by cataloguing research gaps, or the places that need more investigation.

Regarding the slip border condition, Section 3 explores stagnation point flow. The impact of ferromagnetic & viscoelastic factors on the temp& velocity distribution is also discussed in this chapter. The energy-motion equation is a two-dimensional flow that tends to a stretched sheet. The effects of a magnetic dipole on fluid flow are addressed in this chapter.

The idea of free-stream mixed-mode hydrodynamic flow across an inclined plate is introduced in Chapter 4. Also covered in this chapter is the topic of velocity slip conditions with the influence of porous media. Additionally, the impact of the MF& inclination angle has been investigated. As part of the examination into the flow issue, we also consider the influence of permeability on the distribution of temp.

Using the first- & second-order slip effects, Chapter 5 discusses numerical studies of magnetohydrodynamic (MHD) border layer flow. For the physical model of the flow issue, an exponential stretching sheet is taken into consideration. This chapter examines the effects of the radiation parameter on the Nusselt number & Wall shear coefficient.

Chapter 6, the last chapter, covers the topic's conclusion & its potential future applications. Our study findings & a discussion of the topic's prospective future scope are presented in this chapter. Both the broad & specific meanings of the phrase "future scope" are used in relation to our problem. The bibliography included at the end of the thesis contains not only all of the research articles that were used in the study, but also a few more significant papers & books.

REVIEW OF LITERATURE

2.1 INTRODUCTION

2.1.1 Research background

Unpredictability has characterised the early scientific successes in fluid mechanics. Over the course of the 18th & 19th centuries, basic discoveries started to trickle out. In order to overcome flow challenges, early societies developed solutions such as irrigation systems & sailing ships propelled by oars. Archimedes (285-212 B.C.) created two rules that are applicable to both floating & submerged objects: the law of buoyancy & the parallelogram law for vector addition. The mass conservation equations for one-dimensional constant flow were put up by Leonardo da Vinci (1452-1519) & were tested using various means such as waves, hydraulic leaps, jets, etc. Newtonian fluids were defined by the ideas of linear motion & viscosity laid forth by Sir Isaac Newton (1642–1727). At first, he proved the concept of frictionless flow. The issue of frictionless fluid flow was solved by a number of 18th-century mathematicians, such as Pierre-Simon Laplace, Jean D'Alembert, Leonhard Euler, & Joseph Louis Lagrange. Leonhard Euler (1707–1783) offered two versions of the equation of motion: one with differential & one with integral. But there are limits to the assumptions of fluids without friction when it comes to real-world issues. Overcoming the limitations of the ideal fluid assumption, D'Alembert proved that the body experiences zero drag when it comes into contact with a frictionless fluid. Tests of this theory led to the creation of hydraulic models & data on flows in pipes, waves, open channels, & turbines.

By the late 1800s, theoretical hydrodynamics and experimental hydraulics had begun to converge. In 1883, Reynolds (1842-1912) examined the famous pipe experiment that screened the importance of the Reynolds number (dimensionless), while Lord Rayleigh (1842-1919) offered the dimensional analysis approach and William Froude (1810-1879) and Robert (1846-1924) gave the model testing procedures. The equations that control motion were updated by Navier (1785-1836) and George Gabriel Stokes (1819-1903) to include Newtonian viscous components; these equations are now called the Navier Stokes equations. Ludwig Prandtl (1875–1953) established the border layer

theory after studying flows of low viscosity fluids like air & water. The following are a few of the most important turning points in the evolution of border layer theory:

When it came to mathematical models that attempted to explain how fluids behaved close to solid surfaces, Pr&tl was the pioneer. He also popularised the idea of a border layer. Research dominated the subsequent activity of Pr&tl. Birch (1920) exp&ed on Pr&tl's previous work by offering a more thorough mathematical description of the border layer. This description included the first models of turbulent border layers. Karman (1930) significantly advanced our underst&ing of turbulent border layers by establishing many important mathematical models for their behaviour. The first all-inclusive textbook on border layer theory was created by Schlichting (1940) &exp&ed upon by Pr&tl, Birch, & von Karman. As a classic in the subject, his textbook continues to see heavy usage even now. New models for describing the behaviour of border layers in high-speed flows were created by von Dongen&Laitone (1960), who made significant advances to our knowledge of these phenomena.

The region of study known as computational fluid dynamics, which began in the 1980s & is still operational today. It was only via the development of numerical methods & high-speed computers that researchers were able to directly solve the equations of fluid mechanics & predict the behaviour of fluid flow in complex systems. As a consequence, accurate predictions of the behaviour of fluid flow were made, new insights were gained, & border layer theory underwent a revolution. In the process of solving a number of mathematical problems that include viscous flows, the concept of the border layer has shown to be of great assistance. This idea has been a significant benefit to the area of fluid mechanics ever since it was first introduced in the early 1900s. Tani (1977), Keller (1978), Shampine& colleagues. (2000), &Kierzenka&Shampine (2001) provide an overview of border layer theory as well as a numerical approach for solving equations that are associated with border layers on their respective publications. Sakiadis carried out a theoretical investigation into border-layer flow on surfaces that had an infinite length in 1961. He then compared the findings of this investigation to those obtained for continuous solid surfaces. We were able to get the DE for the border layer as well as the integral momentum equations for the surfaces. A demonstration of the application of DE & the theory of differential border layers to the motion of continuous surfaces was provided by the result. Due to

the fact that the borderconditions for the surfaces were different in each of the two cases, the solution to the DE of motion will provide a significant difference between the two examples. In addition to this, he examined the border layer flow equations for both laminar & turbulent flows on moving surfaces that were uniformly flat & cylindrical. The behaviour of the laminar border layer was investigated using both a numerical technique & an integral method, while the behaviour of the turbulent border layer was investigated using just an integral method. He formulated the equations that describe the thickness of the border layer, the thickness of the displacement, the momentum, & the flow under circumstances that are both laminar & turbulent. Skin friction & the equation of thickness are two concepts often discussed. When we investigated the behaviour of border layers on continuous surfaces, we found that it was distinct from the behaviour of border layers on surfaces with finite lengths. The drag on a surface that was flat & had a limited length was lower in a turbulent border layer, while it was higher in a laminar border layer.

Bhatnagar (1961) investigated the Nonlinear fluid model's incompressible two-dimensional flow. In his study, he evaluated the residual viscosity of the border layer using a constant cross-viscosity. In a Newtonian fluid flow, the pressure in the border layer is lower than in a comparable non-Newtonian border layer with the same viscosity coefficient; conversely, as pressure is increased, the non-Newtonian border-layer equations converge to the Newtonian fluid ones. The results proved this. Another finding that followed was that the two fluids' velocity distributions were similar.

The border layer flow & the continuously moving surface were both studied theoretically & empirically for laminar & turbulent flows by Tsou & colleagues. (1967). Analytical solutions provided the findings for border layer velocities, temp distributions, Heat exchange coefficients, & surface friction. Results from studies with laminar flow & agreement In relation to analytical predictions & actual velocity profiles shown that border layer flow across a continuous surface may be physically implemented. There was also strong agreement In relation to the analysis & the experimentally measured turbulent velocity patterns. When it comes to the turbulent border layer, the temp distribution data match up well with the analysis. In addition, the data from the investigations strongly backed up the analytical model of the turbulent flow field. While the results in the velocity field were more widely accepted, the

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experimental information on the distribution of temp in the turbulence field was good & consistent with the analytical predictions.

By using two-dimensional, stable, incompressible, laminar border layer flows, Hansen & Na conducted their research in 1968 to investigate the commonalities In relation to Nonlinear fluid models. The research demonstrates that for any Nonlinear fluid model, there is a similar solution where the stress & strain rate are associated with a continuous arbitrary Expression. This analysis was performed on the case of wedge flow at an angle of ninety degrees. There was also a demonstration of the dimensionless form of quantitative solutions of power law Eyring fluids.

In 1970, Thompson & Snyder were the ones who first explored border layer flow in two-fluid systems. More specifically, they were the ones who incorporated a non-Newtonian fluid in their investigation. These researchers used the same flat plate that is used to inject non-Newtonian fluids in order to investigate the border-layer flow of Newtonian fluids. Through the use of similarity transformations, we were able to reduce the controlling PDEs to ODEs. Following that, we were able to solve these EDES numerically by using the firing method. In conclusion, the data demonstrated that the thickness of the border layer in a non-Newtonian region with a smaller non-Newtonian area increases with increasing mass injection while maintaining the fluid index at the same level. When the mass injection is maintained at a constant level & the fluid index value is raised, the bottom portion becomes thinner. There was a contraction of the outer Newtonian area in both instances.

Vleggaar (1977) studied the transmission of momentum & heat to continuous surfaces that are accelerating. This solution was compared to the ones for continuous surfaces with a constant velocity & to the ones for finite surfaces with border layers throughout their length. The energy & conservation of momentum equations were provided in a methodical manner. The two applications he demonstrated were (1) cooling the sheet (2) heat & momentum transfer to monofilament both of which were solved using the computer software SYSSOL, which governs ordinary DE. Due to acceleration, momentum & Heat exchange were high in dimensionally-invariant values on flat surfaces. The problem became more complicated, however, while dealing with cylindrical surfaces because of the cylinder radius's impact. A comparison with a

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cylindrical continuous surface moving at a constant velocity revealed that an accelerating surface transferred more momentum but less heat.

Rajagopal & colleagues. (1986) conducted study on three-grade fluids with incompressible flow from the perspective of an infinite flat porous plate with suction. This plate was the topic of their investigation. The existence theorem was shown by the use of the shooting approach. An application of the perturbation analysis was made in order to investigate the power problem. The convergence of the perturbation solution may not be the optimal choice for fixing some substantial issues unrelated to the perturbation parameter adjustment as it was not immediately apparent. Finally, the issue was resolved by using a numerical technique. The Gears approach was used to integrate the equation from infinity to the plate, & a satisfactory solution was obtained.

The Walters liquid B flow across a stretched sheet was the subject of the investigation that Siddappa & Abel (1986) carried out in their study. The suction effect was something that they made care to take into mind while conducting their examination. They have solved the energy equation as well as the border-layer flow equation, & they have discovered exact solutions to both of these equations for which they have been responsible. Every single one of these equations is referred to by a name that is distinctly its own. Additionally, the formulae for the border layer thickness & the Wall shear coefficient were calculated & examined. Additionally, the methods for calculating & analysing these formulas were found & explored based on the findings of the previous study.

In the year 1986, Timol & Kalthia carried out research on the similarity solutions for Nonlinear fluid models, which are fluids that are independent of time. The incompressible border layer flows in three dimensions were the primary focus of their study. The results of this study demonstrated that the similarity solution is there for each & every model of Nonlinear fluid models that may be regarded comparable. One of the most major limitations of this study was the assumption that the stress & strain rate were continuous Expressions. This was the most critical drawback.

In their study from 1989, Dandapat & Gupta discussed the border layer approximation of incompressible second-order fluid flow on the stretched elastic plane surface. In addition to that, they spoke on the implications of this approximation. It was

discovered that the amount of friction that exists between the skin & the surface decreases as the elastic parameter (kl) increases. Based on the analysis of transfer of heat for this flow, it has been determined that the temp rises at that particular location when the value of

(which was designed for a certain Pr & tl number) grows. The reasoning behind this is predicated on the assumption that the temp of the walls & the surroundings around them do not fluctuate.

Hassanien & Gorla (1990) derived the border layer equations by examining the characteristics of heat transmission in incompressible, micropolar, steady fluid flow over a stretched sheet flows. This was done in accordance with the theory of micropolar fluids that was presented by Eringen (1972). For the purpose of describing fluid flow, transfer of heat qualities, & major parameter dependence, mathematical solutions were taken into consideration. In addition, the values for the thermal functions, the angular velocity, & the velocity are provided. We were able to ascertain the rate of transfer of heat as well as the skin friction factor by using this knowledge. It was found that the mass transfer rate had an effect on both the rate of transfer of heat & the friction factor. This is because suction has the capacity to establish uniform temp & velocity profiles, which in turn reduces the gradients that are present in these variables. An important element that plays a role in heat transmission is the power law index, which is also referred to as the power law exponent in certain circles. The Pr & tl number is increasing, which results in an increase in the rate of heat transmission. By raising the Pr & tl number, one may produce a thinner thermal border layer, which in turn leads to a reduction in the transfer of heat resistance.

The heat exchange and flow of a viscoelastic fluid in a parallel free stream over a continuous and flat surface was shown by Hassanien in a 1992 manuscript. The boundary layer posed new problems and offered new answers when contrasted with movement on a flat surface with a fixed length. The flow on solutions with continuous surfaces is defined by the velocity ratio and the relative velocities of the continuous plate and free steam, but in the case of border layer flow, the only factor that determines flow on a flat surface of finite length is the velocity differential. Numerical solutions allowed us to extract and tabulate the distribution profiles of velocities and temperatures over a wide range of viscoelastic parameters. We were able to examine

the data because of this. Furthermore, fluid flow solutions and heat exchange characteristics may be studied with the use of numerical solutions. If the variations in Pr & tl , Reynolds, and velocity are all equal, then when U_w is larger than U_∞ , the surface heat exchange rate and surface friction become important. Unlike when $U_w = U_\infty$, this is not the case here.

In their investigation of the phenomena, Wu and Thompson (1996) used both experimental and computational approaches to evaluate the effects of Reynolds numbers on the lift and drag of flat plates in non-Newtonian fluids that are shear-thinning. This was done in order to better understand the phenomenon. They were able to precisely estimate the lift and drag coefficients by using FEM code, which led to promising experimental experiences. This was a consequence of the fact that they predicted the coefficients. As a consequence of the findings of their inquiry, the Reynolds numbers were calculated. These numbers, which are a function of the shear-thinning fluid index, identify situations in which the border layer approximation would not be sufficient.

Timoshin (1997) conducted an investigation on the stability of Nonlinear fluid models with border layers by allowing them to be spread out across a thin sheet of different fluids. In order to analyze the two-fluid system that he was considering, he used the triple-deck theory. Using this approach, a similar examination into thin layers of Nonlinear fluid models was carried out. The inquiry focused a significant amount of emphasis on the instability that emerges in two-fluid flow systems.

In his work from 2001, Ariel tackled the problem of a second-class liquid flowing in a laminar & steady manner across a sheet that was stretched radially. The results of this research demonstrate that fluid viscoelasticity & differential equation order impact both the border value issue & the process of forming border conditions. There was a solution for every possible value of the viscoelastic parameters, according to the research. In addition, we discover asymptotic solutions for high viscoelastic parameters, as well as perturbation solutions for low viscoelastic values. Any of these options will be effective when the viscoelastic parameters are considered to be high.

Research conducted by Xu & Liao (2009) focused on the behavior of Nonlinear fluid models, also known as power-law fluids, when they pass through a border layer while

traveling over a stretched sheet. When considering the stretched sheet, the velocity distribution was represented by the equation $U(x) = Cx^m$, while the temp distribution was represented by the equation $T_x = T_\infty + Axy$. The solution of the directed global self-similarity equations by the use of the homotopy analysis method has resulted in the discovery of the appropriate analytical approximations. According to the findings, the rate of Heat exchange improves & the Wall shear coefficient falls in fluid flow as the power-law index grows. This is the case in fluid flow.

Patil&Timol (2012) The objective of studying the border layer flow of a non-Newtonian fluid with a power-law distribution was to assess 3-dimensional laminar unsteady flow across a flat plate. An entire procedure including the two-parameter groups is used to transform the controlling PDEs & border conditions into the non-linear ordinary differential equation form. The requirements for the similarity solution in the present flow scenario have been established after giving serious consideration to the requirements of the similarity criterion. Subsequently, the most general solution that is conceptually comparable is created.

Within the scope of their study from 2015, Khan & Hashim examined the non-Newtonian two-dimensional border layer flow of fluids. Utilising Runge-Kutta in conjunction with the shot approach allowed for the numerical solution of the DE that governed the system to be obtained. The thickness of the momentum layer rose as the value of the power-law index increased, whereas the thickness of the thermal border layer dropped. Furthermore, it was shown that as the Weissenberg number is increased, the fluid magnitude for shear-thinning fluids drops, but the fluid magnitude for shear-thickening fluids grows. There is a drop in the coefficient of local skin friction as the We number grows. In contrast, raising the We number results in a drop in the Nusselt number, while increasing the Pr&tl number results in a reduction in the thickness & temp of the thermal border layer.

An example of a mathematical simulation of the three-dimensional HNFs model was constructed by Devi & Devi (2016). Furthermore, S. Zhang & colleagues. (2021) conducted an investigation which focused on the properties of $Al_2O_3 \rightarrow CuO$ /water HNF molecules. An increase in the Nusselt number of HNFs is something that they make sure happens. The article by Armaghani& colleagues. (2021) investigates the

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impact of MF & heat source/sink parameters on the flow of a mixed convection Al_2O_3 Cu/water hybrid nanofluid. They made the discovery that a heat sink parameter & a MF angle of 1800 give an effective rate of Heat exchange; this was a significant finding.

In a study that was conducted by H. Waqas & colleagues (2022), it was discovered that the velocity & temp fields improved when the particle concentration reached higher levels. In the next step, HNFs are used to conduct a speedy investigation of a border layer flow under a variety of conditions. A number of citations are included, including those from Iqbal & colleagues. (2017), Usman & colleagues. (2018), Khashi'ie & colleagues. (2020), M. I. Khan & colleagues. (2020), & Waini, Ishak, & Pop (2020).

Recently, research into border layer flow has shifted its focus to include ternary Nano-objects in base fluids. An improvement in thermal performance of up to 14.32% was seen by Sundar & colleagues. (2021) when they research the Heat exchange of a trihybridnanofluid laminar flow including rGO (reduced graphene oxide), Fe_2O_3 , & TiO_2/EG . After numerically approaching the thermal performance of Thnfs under various flow parameters on a stretched sheet, Ramesh & colleagues. (2022) & Manjunatha & colleagues. (2022) found that Thnfs outperformed hybrid nanofluids in terms of thermal distribution & applicability. investigated trihybridnanofluids' three-dimensional flow in relation to their dual stretching & various geometrical configurations. They deduced that, given stretching parameters, Thnfstemp should decrease. The influence of Soret & Dufour variables on tri-hybrid nanoparticle-containing pseudoplastic fluids was investigated by Hou & colleagues. (2022). Using Non-Fourier's law, Algehyne & colleagues. (2021) studied ternary nanofluids flowing past a porous surface & discovered that the flow motion is restricted & greater heat is produced in the case of Thnfs due to power-law, porous medium, & Forchheimer number. The thermal conductivity of the ternary fluid ($\text{Al}_2\text{O}_3 + \text{ZnO} + \text{MWCNT} \rightarrow \text{EG}$) was experimentally investigated by Bindu & Joselin Herbert (2022). They found that when the quantity of Nano-objects increased, the thermal conductivity increased & the viscosity decreased. Moreover, these references allow one to go into studies concerning the flow of trihybridnanofluids using different setups, fluid parameters, & computational approaches. Nasir & colleagues. (2022) & Mousavi & colleagues. (2019).

Stretching surfaces problem involving Williamson nanofluid for the purpose of studying pseudoplastic flows, Williamson (1929) developed a model & provided experimental evidence to back it up. Noting that pseudoplastic flows vary significantly from viscous flows, he went on to explain their significance. Nadeem & colleagues. (2013) used the HAM technique to examine a Williamson fluid flowing over a stretched sheet. They discovered that the Wall shear coefficient & velocity profiles dropped as the Williamson parameter rose. In the presence of an electric field, heat radiation, & a MF, Williamson fluid was observed by Hayat, Shafiq, & Alsaedi (2016) as it moved over a stretched surface that was prone to wobbles. It is possible for us to investigate the impact that physical elements have on velocity & temp by solving the governing equations, which are now known as ordinary DE, using analytical methods. In addition to this, we demonstrate & investigate the Wall shear coefficient as well as the local Nusselt number. In order to achieve one of the outcomes that they desired with larger Williamson parameter values, they wanted to see a decrease in wall shear stress.

An investigation on the transmission of heat and mass across a porous extended sheet was carried out by Shawky and colleagues in 2019. The investigation was carried out using a Williamson non-Newtonian fluid that was exposed to a MF, Brownian motion, and thermophoresis processes. The integration of the shooting strategy with a fourth-order Runge-Kutta method was used in order to discover that equations had a numerical solution. Both the porosity and Williamson parameters are responsible for determining the rate of transfer of heat at the surface that has been stretched. Both of the characteristics are shown to increase in size when the surface porosity increases, as shown by the findings. The primary focus of the research that Nadeem and Hussain conducted in 2014 was on the impact that nanoparticles have on the flow of a Williamson fluid in two dimensions over a stretched sheet. According to the authors of the research, heat transmission is highest near to the wall and slightly diminishes away from it. This is the case regardless of the physical qualities that are present, such as the Prandtl number, the Brownian motion parameter, or the Lewis number.

Salahuddin & colleagues. (2019) conducted research on the two-dimensional Williamson nanofluid flow in permeable space that occurs as a consequence of a surface that has changing thickness. The Darcy-Forchheimer model of porous media,

the Cattaneo-Christov assumptions of heat & mass flux, & the exponential stretchable sheet were the tools that they used in order to construct flows. With the use of the bvp4c approach, the researchers were able to solve the equations after first converting them to ODEs. There was a decrease in velocity profiles that was seen when the Williamson & inertia parameters were increased.

It was discovered by Immaculate & colleagues. (2016) that the MHD flow of Williamson nanofluid over a porous surface is influenced by the time-dependence of the flow. This was a significant finding. In order to discover a solution to this issue, the HAM is used once the non-periodic & periodic components of the ODE model of this problem have been separated from one another. Although the Grash of number & the Brownian motion parameter both contributed to an increase in the velocity of the fluid, the findings demonstrated that the thermophoresis parameter was responsible for a drop in the velocity as its value grew. This was the case despite the fact that both of these parameters contributed to an increase in the velocity.

In their study on entropy optimisation in unstable three-dimensional MHD non-Newtonian nanofluids, Nayak& colleagues. (2020) discovered that when the magnetic & unsteadiness parameters were increased, the velocity profile decreased. Shateyi&Muzara (2020) conducted a numerical study in which they investigated the heat &Transport of species of MHD Williamson nanofluid over a stretched sheet in a flow that was dependent on the passage of time. They came to the realisation that the velocity & momentum layer thickness decreased in proportion to the rise in the unsteadiness parameter. In their study of Williamson nanofluids' unsteady border layer flow, Kebede& colleagues. (2020) discovered that when the parameters of unsteadiness, porosity, &MF increase, the velocity profile decreases. Considering the three-dimensional Casson Cross micropolar nanofluid, Nayak& colleagues. (2020) analysed the decreasing behaviour of the velocity profile with regard to the Weissenberg number. Both the Muthamilselvan& colleagues. (2019) & the Rana& colleagues. (2019) experiments made effective use of Williamson micropolar fluid. The Williamson parameter, as the researchers found out, raises the Wall shear coefficient& slows down fluid flow. The peristaltic movement in blood, medicine, food mixing, and studying the chime motion in the gut are just a few of the many engineering, medical,

and industrial applications of Williamson fluids. Assoc. Vajravelu. referenced are (2012) and Vasudev& colleagues (2010).

A large number of researchers have developed numerical models & obtained results for the convection flow of rheological fluids; for example, Anantha Kumar & colleagues. (2020b), Anantha Kumar & colleagues. (2020a), K. A. Kumar & colleagues. (2020), M&al& colleagues. (2020), M. Khan &Alzahrani (2021), Khan &Alzahrani (2021). These studies have discovered that the convection parameter has a tendency to increase the fluid velocity. A. Hamid & Khan (2018), Rasool& colleagues. (2019), Loganathan&Rajan (2020), A. Kumar & colleagues. (2020), Krishna, Swarnalathamma, &Chamkha (2018), Krishna, Bharathi, &Chamkha (2018), & A. Hamid & Khan (2018) are quoted about the importance of Nonlinear fluid model in different border layer issues with complicated restrictions. Recent research by Haider& colleagues. (2021), Shed & colleagues. (2021), &Alshehri& Shah (2022) examines the hybrid Williamson fluid with a single-phase nanofluid model, confirming that hybrid nanofluids are more efficient than nanofluids.

2.2 BORDER LAYER FLOW OF MICROPOLAR NANOFLUID PAST STRETCHING SURFACES

Micropolar fluids, in which nanoparticle micro-rotation is crucial, were first introduced to the scientific community by Eringen (1966) and Eringen (1972). Researchers Peddieson and Mcnitt (1970) and Gupta and Gupta (1977) looked examined micropolar fluid flow across an extended surface and its impact on heat transmission, respectively. Researchers Qasim and colleagues. (2013) and Turkyilmazoglu (2016) discovered that the velocity profile grows as the material (micropolar) indicator rises while studying the transfer of heat characteristics of micropolar fluid.

Research by Patel and colleagues. (2019) focused on the microstructure and inertial properties of the substructure particles in a micropolar nanofluid (Fe₃O₄/water) MHD flow. It is necessary to use the appropriate algorithms to transform the controlling nonlinear dimensional equations into dimensionless expressions before HAM may solve them. Acceleration and microrotation field were found to be decreasing when the intensity of the MF was raised.

The MHD & Darcy-Forchheimer models were used by Shah & colleagues. (2020) in order to explore the effects of heat radiation, suction, & injection on a micropolarferrofluid that was travelling across a sheet that was both shrinking & stretching. The magnetic & electric fluxes that originate from the outside environment are also taken into consideration. In the scenario involving the stretched sheet, the researchers discovered that the velocity field dropped as the values of the porosity, inertia, & magnetic coefficients grew throughout the experiment. No matter whether the sheet is being stretched or shortened, the temp profile exhibits a rising trend. This is the case regardless of whether the values of the thermal radiation parameters are increasing for positive or negative values of S . Alterations to the Pr number & the thermal relaxation parameter also result in changes to the temp profile.

In their study, Jeevandhar & colleagues. (2021) examined the flow characteristics of a micropolar nanofluid containing Fe_2O_3 nanoparticles suspended in a water-& ethylene glycol-based base fluid. Researching flow characteristics & the effects of changes in micropolar parameters & nanoparticle volume concentration was their main focus. The study's results demonstrated that the skin friction coefficient increased noticeably as the volume concentration of nanoparticles & micropolar particles increased. It was discovered that this behaviour was caused by the suspended particles' greater resistance. Because of this obstruction, the fluid's effective viscosity rose, which strengthened the bond between the fluid & its surrounding surface. So, more skin friction resulted from higher shear stresses at the surface, which were caused by higher particle concentrations.

Shehzad & colleagues. (2020) performed related research in which they examined the influence of porosity & a MF on a viscoelastic micropolar fluid. According to their findings, particle rotation has a major impact on the dynamics of the flow. A rise in the fluid's velocity was the end result of improved momentum transfer brought about by the spinning activity of micropolar particles. Thanks to the particles' very fast deformation, this was achieved. But the presence of an MF caused a Lorentz force to counterbalance the flow motion, resulting in a decrease in the total fluid velocity. It seems that viscoelasticity, micropolarity, MF, and porous media all play a part in regulating and influencing the behaviour of fluid flows.

Bhattacharyya & colleagues. (2012) noted that high material factors result in high mass suction values, & they also noted that the first & second solution scenarios for micropolar fluid flow affected by radiative heat & mass transpiration showed mixed behaviour for Surface friction, couple stress, & Nusselt number. In 2014, Turyilmazoglu, a researcher from Turkey, investigated the flow dynamics of a micropolar fluid passing through a thin, porous sheet. In a more advanced approach, Turyilmazoglu (2017) determined the precise answer to the issue of micropolar fluid flow across a porous extended heated/cooled surface driven by an MF and a combination of convective and radiative heat exchange. Using mass transpiration as an example, Aslani and colleagues analysed micropolar fluid flow. (2021) discussed radiation and skewed MHD effects. They found that increasing micropolar parameters enhanced microrotation profiles & velocity in both injection & suction cases. Several researchers study micropolar fluid using the one-phase nanofluid model, which accounts for the thermophysical properties of Nano-objects & conventional fluid separately.

In their study, Nabwey & Mahdy (2021) examined the behavior of a micropolar dusty hybrid nanofluid that was subjected to border conditions such as a PST & a PHF. The effects of suction on the fluid's flow & thermal characteristics were highlighted in their study. The findings showed that in both the PST & PHF cases, a significant drop in fluid velocity was produced by raising the suction parameter. This drop was associated with the elimination of border layer fluid particles, which in turn reduced momentum transmission. Micropolar particles' angular velocities rose in turn with increasing suction, suggesting that improved coupling in relation to microelements & surrounding fluid amplified microrotation effects.

The thermal & flow characteristics of a micropolar hybrid nanofluid were studied by Gumber & colleagues. (2022) using CuO & Ag Nano-objects dispersed in water. Their research allowed them to establish the relative roles of suction & injection in determining the heat flow rate. The results showed that when it came to improving the thermal flow rate of the micropolar hybrid nanofluid, the injection effects were more important than the suction effects. Injection increased heat transmission by making the thermal border layers thicker by supplying the system with additional energy. The main

restriction to heat diffusion, however, was suction. This study highlighted injection as a key controlling parameter for micropolar hybrid nanofluid flows' thermal increase.

Many researchers in the field of micropolar fluid dynamics have recently expanded our understanding of this topic by delving into the effects of various controlling variables. (N. Abbas & colleagues., 2022, Bilal & colleagues., 2022, Ch&Sood, 2022), & Ali & colleagues. (2022) reviewed the literature on micropolar fluid flow & found that it is affected by factors like thermal radiation, nanoparticle concentration, suction & injection, MF intensity, & porous media. To find out how these things affect micropolar fluid movement, researchers set out to conduct these investigations. Micropolarity & external forces significantly affect microrotation profiles, temp, & velocities, according to the results of these studies taken as a whole. Therefore, in high-tech engineering & manufacturing, these elements provide efficient methods for regulating flow behaviour & Heat exchange efficiency.

2.3 FLOW ANALYSIS OF POWER-LAW NANOFLUID OVER EXTENDING SURFACES

With the help of the theory of border surface flow, Schowalter (1960) was one of the first researchers to describe power-law fluids. It was proved by him that the border layer equations that regulate power-law fluid motion allow for self-similar solutions, given that the external velocity distribution follows the form $U_m(x) = x^m U_e$. He did this by ignoring the effects of body forces. The formula $(x)=x^m$ is used to calculate the distance along the surface of the body, where x represents the distance. The results of his investigation laid the theoretical groundwork for Falkner Skan type flows in non-Newtonian power-law fluids and demonstrated that the presence of similarity solutions is highly dependent on the external velocity profile that is imposed.

In a research that was complimentary to the one that was conducted by Acrivos & colleagues in 1960, the border layer behavior of power-law fluids was investigated for the particular situation when $W = 0$ & $m = 0$. This case corresponds to flow over a flat plate with a free-stream velocity that is uniform. The emphasis of their work was on border layer flows of the Falkner–Skan type, & they established the existence of self-similar solutions for power-law fluids under these circumstances. The findings not only expanded the concepts of traditional flat-plate border layer theory to include power-law

fluid behavior, but they also gave significant insights into the ways in which non-Newtonian rheological features impact the evolution of border layers.

One of the most important contributions that Lee & Ames (1966) made to the study of non-Newtonian border layer flows was the derivation of similarity solutions for both Eyring viscous fluids & power-law fluids. This was accomplished by the use of a group theory technique. Analytical & semi-analytical solutions were made possible as a result of their research, which used symmetry analysis to systematically reduce the governing PDE of border layer flow into ODE. Even for sophisticated rheological models that stray from Newtonian behavior, they showed how the invariance qualities of the governing equations may be utilized to generate similarity variables by using group transformations. This was done in order to acquire similarity variables. A comprehensive & unified framework was offered by this method, which allowed for the investigation of a broad variety of Nonlinear fluid model flows within the context of border layer theory.

In addition, Lee & Ames (1966) developed generalized DE that are applicable to a variety of flow configurations. These equations may be used to stretched surfaces & external flow fields that have changing velocity distributions. The manner in which these generalized equations are tied to the particular instances of power-law fluid flows under a variety of border & physical circumstances was thoroughly explained by them. Through their investigation, they were able to shed light on the circumstances that are necessary for the existence of similarity solutions & bring to light the impact that rheological factors have on displacement & shear stress distributions. Their work not only extended classical border layer solutions to Eyring & power-law fluids, but it also provided the framework for forthcoming research that would use symmetry & similarity approaches to study complicated non-Newtonian flow issues in engineering & applied sciences. This was accomplished by laying the groundwork for their work.

In order to determine if Nonlinear fluid models may reduce drag, Thompson & Snyder (1968) injected fluid at the surface into border layer flows involving power law fluids. The answer, which changed over the range of injected fluid masses, states that the coefficient of Surface friction monotonically decreases as the fluid index rises. Nevertheless, while the mass injection rate of fluid remains constant, the skin-friction

coefficient decreases at a faster rate with a lower fluid index value. With respect to changes in momentum & displacement thickness, further findings were drawn for a variety of fluid injection rate ranges & fluid index values.

&ersson&Toften investigated the potential of numerically addressing the border layer flow issue for power-law fluids, which are Nonlinear fluid models, in their article that was published in 1989. They simplified the techniques & went over the several approaches that may be used to solve the issues. Before the Keller box approach was used, the governing PDE was converted into a non-linear ODE. When it comes to addressing border layer concerns with Nonlinear fluid models, the most important thing that can be learned from this study is that the Keller box technique is preferable to exactness in the power-law viscosity model.

A study conducted by Y. Zhang & colleagues (2017) investigated the heat & mass transport of non-Newtonian fluids by using the power law model under a variety of slip situations & MF parameters. After conducting their study, the researchers discovered that the power law index gets more complicated when the Nusselt number & skin friction both rise. When the MF parameter was raised in power-law fluid study conducted by Ahmed & colleagues. (2016) & Lin & colleagues. (2015), it was discovered that the Nusselt number & velocity both decreased. The effects of entropy & transfer of heat on power-law fluids were investigated by Ullah& colleagues. (2021), who discovered that the latter became more significant as the Pr number reached higher values. For a more in-depth examination of power-law modelling of non-Newtonian fluids in a variety of settings, please refer to the works of S. Hussain & colleagues. (2021), Elfeshawey&Waheed (2022), &Dey&Mahanta (2022).

According to Moghadassi& colleagues. (2015), who carried out a numerical investigation of the flow behaviour of water-based nanofluids that included A1203 & A1203 + Cu, the Nusselt number of the hybrid material was 4.73 percent greater than that of the A1203/water solution. This was the conclusion reached by the researchers. In conclusion, Chamkha& colleagues. (2017) have used the finite difference approach in order to investigate the characteristics of the hybrid nanofluid composed of A1203 & Cu/H₂O, which ultimately leads to improved transfer of heat. Ghadikolaei& colleagues. (2017) conducted research on the TiO₂ stagnation point flow & discovered

that platelet-shaped nanoparticles were the most significant. This was the conclusion reached after taking into consideration the impacts of nanoparticle form factors & the presence of copper/water simultaneously. Following that, Ghadikolaie & colleagues. (2018) conducted an investigation to determine whether or not a non-Newtonian nanofluid that included a mixture of water, ethylene glycol, & a hybrid of TiO₂ & CuO may potentially enhance the temp characteristics. According to the findings of their research, L. Zhang & colleagues (2022) investigated the impact that MF parameters play in minimising fluid motion in hybrid fluid flows that include tantalum & nickel nanoparticles. Research conducted by Waini, Ishak, Gros, an, & Pop (2020), Yasir & colleagues. (2022), Rosca & Pop (2021), & Haq & colleagues. (2022) has made it feasible to explore the flow & transfer of heat features of hybrid nanofluids. These nanofluids are composed of both Newtonian & non-Newtonian components.

2.4 GAPS

There have been a lot of research on Newtonian fluid behaviour, but most of them have used oversimplified analytical models that can't handle complicated geometries, border conditions that may change, & flow situations with more than one dimension. The effect of processing limits & numerical precision on the reliability of simulation findings is often disregarded in the existing literature. Furthermore, there may be disparities in practical applications since many previous efforts fail to adequately check numerical solutions against experimental or real-world data. When it comes to addressing issues involving Newtonian fluid flow under varied circumstances, there is a dearth of thorough research that systematically evaluate various numerical approaches. These techniques include finite element, finite difference, & finite volume methods. These deficiencies call attention to the need of a stronger numerical method that guarantees precision, consistency, & relevance to practical fluid flow systems.

2.5 THEORIES

The Newtonian Fluid Theory, which serves as the basis for this inquiry, postulates that the viscosity of a fluid is not dependent on the shear rate that is applied to it. In addition to this, the study makes considerable use of the Navier-Stokes Equations, which are important to the field of fluid dynamics & describe the motion of substances in viscous fluids. Additionally, the use of heat & mass transfer theories is utilized in

order to conduct an analysis of the thermal energy & mass transfer that occurs inside the fluid flow. By using Numerical Analysis Theory & applying techniques such as finite difference, finite volume, & finite element approaches, these difficult DE may be discretised & solved computationally. This is possible via the use of numerous techniques. When everything is taken into consideration, these theories provide a good foundation for describing Newtonian fluid flow, simulation, & understanding transfer processes.

2.6 FRAMEWORK

The conceptual foundation of this study is provided by mathematical modelling of Newtonian fluid flow and numerical solution of the equations that regulate the flow. Continuity, momentum (Navier-Stokes), and energy equations are some of the governing equations that are constructed and used in the framework for the purpose of identifying the physical system. After that, the appropriate numerical methods are used in order to discretise these equations in order to facilitate computational analysis in a more straightforward manner. In addition to include essential variables such as velocity, pressure, temp, and viscosity, the framework takes into consideration border and starting conditions that are relevant to the fluid system that is being investigated. A technique that involves solving the numerical model in an iterative manner and testing it against benchmark numerical data or analytical solutions (if they are available) is described here. In conclusion, the framework's objective is to improve process design and fluid system analysis by precisely anticipating the transfer of physical properties in Newtonian fluid flow. This will be accomplished by using the framework.

SLIP EFFECTS ON UNSTEADY MIXED CONVECTION NANOFLUID FLOW OVER STRETCHING SURFACES

3.1 INTRODUCTION

Ferrofluids become very magnetized when exposed to a MF. Some examples of these fluids include water, kerosene, & heptane. A ferrofluid's mechanics may be influenced by strong MFs. In ferro hydrodynamics, non-conducting liquids with magnetic properties are often studied. Ferro hydrodynamics is a common topic in the study of magnetic properties in nonconducting liquids. By passing a MF across a ferrofluid, its flow may be characterized. How quickly heat moves In relation to border layers on a stretched surface is a topic that many researchers have looked at. Because of its practical technical implications, this issue has attracted a lot of interest in the areas of refrigeration & cooling, paper manufacturing, & structural design. In 1970, Crane & colleagues. conducted the seminal work on the stretching sheet in laminar viscous fluids. According to research by Elbashbeshy & colleagues. (2004), the Heat exchanges in a radiation-induced unstable laminar fluid flow across a stretched surface was examined. (Chakrabarti & colleagues., 1979) examined the temp distribution of fluid flow through a stretched sheet when suction & injection were applied.

The unstable viscous nanofluid flow across a stretched sheet was studied by Grubka & colleagues. (1985). The study examined the relationship between the viscosity coefficient & non-Newtonian MHD fluid flow (Ellahi & colleagues., 2013). As the Hartmann number drops, the velocity of pseudoplastic fluids falls as well (Khan & colleagues., 2016). Hayat & colleagues. (2015) offered research on the transfer of heat & mass through an exponentially stretched surface, taking into account the impact of the velocity slip condition. Within the scope of their research, Narayana & colleagues (2012) investigated the impact that the radiation parameter had on the surface-border velocity gradient that was brought about by the thin film. An investigation of the influence of heat radiation on the flow of convective nanofluids across a moving surface was carried out by Abdel & colleagues. in 2007. The issue of how a MF that is routinely provided affects the flow of a viscous fluid is discussed in the references

(Abdelwahed& colleagues., 2017) & (Abdel-Wahed M. &Akl M., 2016). By studying fluid flows

that were neither Newtonian nor non-Newtonian, Chiam& colleagues. (1998) were able to compute the Heat exchange coefficient.

In their 2010 study, Khan and Pop and colleagues. examined how the flow near the stretched surface was affected by the Brownian motion coefficient and thermophoretic parameters. (Raju and colleagues., 2016) studied the temp distribution in multi-homogeneous fluid dynamics (MHD) as a function of velocity slip condition with respect to the stretched permeable surface. A comparable study was carried out by (Mustafa and colleagues., 2013) using unstable two-dimensional flow over a stretched sheet. In their 2014 study, Aziz and colleagues. delved into the issue of mixed convection flow of viscous fluids with velocity slip effect. (Dutta and colleagues., 1985) computed the temp distribution in nanofluid flow as a function of MF applied to a stretched surface. In their 2010 study, Hayat and colleagues. assessed the numerical solution of border layer flow close to the stagnation point, as well as the impact of heat radiation on transfer of heat. (Ishak and colleagues., 2009)

The behaviour of mixed convection & other unstable flows over a stretched vertical surface was the subject of these theoretical considerations. In the research that was carried out by Ibrahim & colleagues. (2013), the effects of a MF & velocity slip on nanofluids that transport heat across a stretched porous surface were investigated. In addition, Andersson& colleagues conducted a research in 1992 that addressed the issue of viscoelastic fluid flow over a stretched surface while the surface was subjected to a MF. The border layer flow issue that is caused by a sliding plate was addressed by Elbashbeshy& colleagues. (2014), who discovered a mathematical solution to the problem. Bachok& colleagues. (2012) conducted research to determine how the presence of a MF influences the flow of stagnation point fluid through a sheet that is contracting. Sheikholeslami& colleagues. (2014) conducted a study in which they investigated the heat transport characteristics of an electrically conductive ferrofluid. Specifically, they focused on the heat production & absorption properties of the ferrofluid.

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A study was conducted to investigate the impact that a surface that is continually expanding has on the heat transform of laminar fluid flow. Both Chen & colleagues. (1998) & Faraz & colleagues. (2012) investigated the influence of temp dispersion on the viscoelastic characteristics of fluids. The stagnation point flow for a viscous fluid was determined by Abbas & colleagues. (2014), who took into consideration the presence of unstable flow conditions. The Maxwell fluid characteristics were used by Mukhopadhyay & colleagues. (2012) in order to implement the transmission of heat & mass across a stretched sheet. It was published in 2013 by Bachok & colleagues. that they have conducted a thorough investigation into the topic of fluid flow in two dimensions. In 2005, Partha & colleagues conducted research on the flow of mixed convection across a sheet that was stretched vertically. The numerical solution for the heat transform of nanofluids was presented by Khan & colleagues. (2012). This solution was obtained via the use of the finite difference technique. A significant amount of research on border layer flow is focused on the unstable Maxwell flow that occurs under the influence of homogeneous & heterogeneous forces.

Following up on these investigations, (Mukhopadhyay and colleagues., 2013) examines the Maxwell fluid flow as it relates to suction and injection effects. In their study, Ellahi and colleagues. (2018) looked at how thermal radiation causes border layer fluid flows to generate certain entropies. In their 2016 study, Bovand and colleagues. investigated the flow of MHD fluids via a porous stretched surface. A constant flow cannot be achieved without MFs, according to this research.

Researchers looked examined how various base fluids' heat transport was affected by the stretched sheet. A more accurate numerical method for transfer of heat based on nanofluid properties is offered. In this work, Jian and colleagues. (2015) examined how mixed convection fluid flow is affected by double diffusion across an unstable spinning media. In order to solve the Prandtl-Eyring fluid flow issue, Hussain and colleagues. (2019) used mathematics. Jafer and colleagues. (2012) tested the effect of Joule heating, MFs, and viscous dissipation on laminar flow of fluids. In their study, Ijaz and colleagues. (2019) examined the flow of ferromagnetic Powell-Eyring fluids using a modified Fourier law and two equal magnetic dipoles. Dust and heat radiation's effects on convective ferromagnetic fluids were thoroughly detailed by Sharma and colleagues. (2005).

The topic of fluid mechanics known as ferro hydrodynamics focuses on nonconducting liquids that possess magnetic characteristics as its primary area of enquiry. Because of the structural similarities it has with a MF, ferromagnetic fluid is an effective instrument for analyzing the magnetic properties of fluid flows.

When looking at border layer fluid issues, most studies have shown that ferromagnetic fluid properties, such as high magnetic domain effects & enhanced heat conductivity, are useful. The properties of a liquid & the magnetism of its constituent particles are combined in ferromagnetic fluids. Given the presence of an EMF. Ferrofluids are repelled by MFs & move in the opposite direction. The application of a MF in the direction of flow increases the flow resistance, which is used to promote Heat exchange using ferrofluid. Here, ferrofluids outperform conventional fluids.

The origins of border layer flow, which have just lately been discovered, have been the topic of a great deal of discussion. Researchers & industry professionals have focused their attention on a wide variety of aspects of heat transport & transformation across a stretched surface. There are a number of applications in the fields of science & engineering for a sheet that is continually changing or expanding, & one of these applications is to force viscous fluids toward the border layer. All of these things paper, automobiles, & medical devices are the end results of common industrial processes. The phrase "mixed convection flow" refers to the cooperative action of free & forced convection. This kind of flow does not occur in isolation. In the event where forced & free convection fluxes are coupled, it is feasible to get mixed convection flow. This kind of movement may be seen in a wide variety of natural & industrial transportation systems, including as nuclear reactors, the ocean, & the atmosphere. The first research to investigate the transfer of heats that occur in nanofluid flows that are caused by convection was conducted by Choi & colleagues. (1995). An investigation was conducted by Panday& colleagues. (2021) to determine the impact that a number of different physical parameters have on MHD convective nanofluid flows that occur on a curved surface. In their study, Revathi& colleagues. (2021) investigated the combined effect of buoyant forces & chemical processes on the flow of viscous fluids. They did this while taking into account the phenomenon of free convection that occurs as a consequence of a curved stretched surface.

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Zhang & colleagues. (2021) found that nanofluids may be used to suction a stretched sheet. The mixed convection fluid flow towards the stagnation point was also investigated in (Ahmad & colleagues., 2020), alongside Heat exchange with radiation effect. In a study conducted by Acharya & colleagues. (2020), the impact of SWCNT & MWCNT Nano-objects on border layer flow over a curved stretched sheet was examined. Navided& colleagues. (2016) calculated the impact of a pressure gradient on a fluid's unstable flow on a curved surface. Researchers should look at mass & heat transport because of higher-order slip border circumstances (Ibrahim & colleagues., 2019). The nanofluid flow issue was numerically addressed using the MATLAB tool, according to Wahid & colleagues. (2021). Awais& colleagues. (2016) looked at how heat exchange in fluids is affected by heat production & absorption.

When the research was released, heat & mass transfer was a subject of discussion that was quite popular due to the wide range of practical applications that the study had. Khan & colleagues. (2020) have found a dual solution to the problem of Walter-B fluid travelling through a stretched sheet for the problem that they were investigating. In the research carried out by Song & colleagues. (2022), the researchers looked at how the flow of fluid is affected by the presence of MFs & chemical processes. The border layer flow pattern has been predicted by a number of research by making use of the many fluid parameters that are subject to change. An investigation that was carried out by Muhammad & colleagues (2022) has thrown light on the manner in which the velocity of a nanofluid spreads out along a vertical plate. It was discovered by Khan & colleagues., 2021 that the precise answer to the problem of peristaltic fluid flow that is caused by pressure gradients was found. In the presence of a porous medium, what kind of an effect can partial order slip have on the Pr&tl fluid? Authors Amanumulla& colleagues. (2019) are the reference. Jhou& colleagues. (2021) conducted an investigation on the MHD flow model applicable to the Casson fluid, taking into consideration the impacts of heat sources & sinks. In order to find a solution to the flow problem, Rosca& colleagues. (2015) conducted research employing dual-solution methods, which included both suction & injection conditions. After that, Ahmed & colleagues. (2021) provided the Brownian motion & thermophoresis effect factors by making use of the Williamson fluid flow model.

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The current corpus of knowledge regarding curved surfaces is also enhanced by this study. looked at the effects of joule heating force on the flow of an incompressible, viscous fluid. This research was crucial in solving the fluid model using curvilinear coordinates. Kumar et al. (2020) looked at how thermal radiation and joule heating affect the heat exchange analysis process. Abbas and coworkers. (2013) studied the effect of dynamic viscosity and found the mathematical solution to a two-dimensional fluid flow issue. Khan and colleagues have brought it up. viscoelastic dissipation and heat production are properties of nonlinear fluid models that are considered to be flowing across a curved surface in (2021). For their analysis, Imtiaz and colleagues turned to an analytical approach. (2016) in an effort to resolve the issue of unstable fluid flow over a curved surface caused by drag. A system with a convective boundary condition was studied for heat transport (Abbas & associates., 2020). In their study, Liu and colleagues examined the distribution of temperatures in a Casson fluid flow. (2021), who also looked at how heat sources and sinks affected the results.

The concept of mixed convection across a stretched & curved surface is the primary subject of this investigation. When using this technique, both the influence of heat conductivity & velocity slip are taken into consideration. A set of mathematical equations is used to explain the problem of the border layer. By performing the appropriate similarity transformation, equations that have been translated into a form that is not dimensional have been transformed.

The major focus of this inquiry is on trying to underst& the notion of mixed convection over a surface that is both stretched & curved. Both the impact of heat conductivity & the influence of velocity slip is taken into account while using this method. In order to provide an explanation for the issue of the border layer, a collection of mathematical equations is used. By completing the proper similarity transformation, equations that have been translated into a form that is not dimensional have been converted.

3.2 MATHEMATICAL FORMULATION

Our focus here is on a ferromagnetic fluid's two-dimensional flow. As the stopping point draws near, the inquiry begins. According to current theories, velocity is unimportant. A MF's effects were also considered in the research using dipoles. Using a MF amplifies the ferrofluid particles' intensity. Because internal friction causes heat

to be created, which in turn causes an increase in temp, we can't assume that a fluid's viscosity is constant. When the temp of the fluid rises, its flow rate also increases. The surface temp of the wall seems to rise as a direct consequence of greater mobility. In addition to the parts already mentioned, this method additionally considers the velocity field close to the stagnation point. At $y = 0$, which is perpendicular to the x -axis, the stretchy surface is taken into account in flow geometry. It is believed that the stretched sheet lies perpendicular to the y -axis. It is necessary to produce a flow that follows the b -axis. At its center is a magnetic dipole. Dominance theory states that two forces of similar size operating in opposing directions are equivalent.

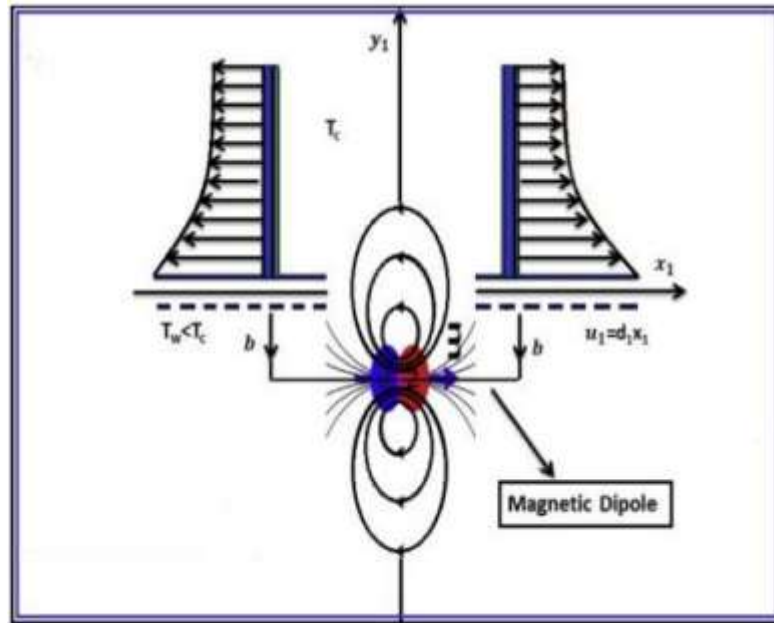


Figure 3.1: Modelling the Flow Physically

Here are the equations for ferrofluid flow & Heat exchange that come from the border layer:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.1)$$

$$\frac{1}{\mu_e} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{M}{\rho} \frac{\partial M}{\partial x} - \frac{1}{\rho \mu_e} \frac{\partial H}{\partial x} + \frac{\mu}{\rho \mu_e} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{g}{\rho \mu_e} (T_c - T_f) \quad (3.2)$$

$$\frac{1}{\mu_e} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \frac{M}{\rho} \frac{\partial M}{\partial y} - \frac{1}{\rho \mu_e} \frac{\partial H}{\partial y} + \frac{\mu}{\rho \mu_e} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{g\beta}{\rho \mu_e} (T_c - T_f) \quad (3.3)$$

$$\frac{c_p}{\mu_e} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) \frac{\partial M}{\partial x} + \frac{T}{\rho} \left(u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} \right) \frac{\partial M}{\partial T} = \frac{1}{\rho \mu_e} \frac{\partial^2 T}{\partial y^2} + \frac{2\mu}{\rho \mu_e} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} \right] \quad (3.4)$$

Please take notice that the x- & y-axis components of velocity are considered to be (i) & (ii), respectively. The sentence lists several terms related to fluids, including "temp," "surface temp," "curie temp," "dynamic viscosity," "magnetic permeability," "specific heat," "MF," & "magnetization," which represent different aspects of the fluid's magnetic properties.

The related border conditions for the flow problem can be represented as

$$u = u_w + S \frac{\partial u}{\partial y}, v = -v_w, T_f = T_w + d_1 x \quad \text{at } y = 0 \quad (3.5)$$

$$u = 0, T_f = T_c + d_2 x, P + \frac{1}{2} \rho(u^2 + v^2) = C \quad \text{at } y \rightarrow \infty \quad (3.6)$$

where u_w and v_w stand for velocity at surface corresponding to x and y axis, respectively, S stands for velocity slip factor, C is constant, d_1 and d_2 stand for dimensionless constants.

In the current endeavour that we are doing, the primary objective is to investigate the dynamics of mixed convective nanofluids. When considering the flow geometry of the issue, a curvilinear stretching surface is taken into consideration first. The influence of slip on the flow field was taken into consideration. The influence of thermal radiation on the temp gradient was another aspect of this investigation that was observed. When measuring a surface that is curved, the radial coordinate (rc) is extracted from the surface itself, while the normal coordinate (rc) is extracted from the surface in the opposite direction. The radius of curvature of the curved surface is represented by the symbol R_c .

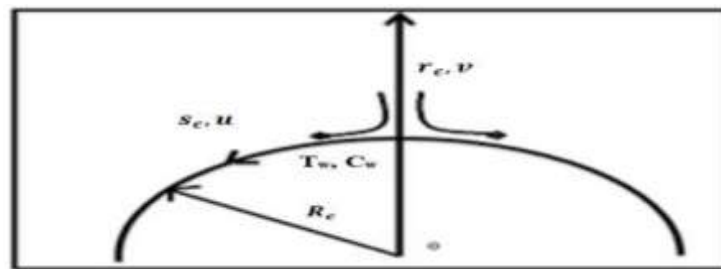


Figure 3.2: Physical model of the flow

Chapter- 3 Slip Effects On Unsteady Mixed Convection Nanofluid Flow Over Stretching Surfaces

T_w refers to the surface temp & C_w refers to the concentration of the surface. T_∞ refers Temp at a long distance from the curved area & C_∞ refers to the concentration at long distance from curved surface.

In order to represent the flow problem of a mixed convective nanofluid, the appropriate border conditions are imposed, & the equations that regulate the flow are as follows:

$$\frac{\partial}{\partial r_c} \{(r_c + R_c)v\} + R_c \frac{\partial u}{\partial s_c} = 0 \quad (3.7)$$

$$\frac{1}{\rho_{nf}} \frac{\partial p}{\partial r_c} - \frac{u^2}{r_c + R_c} = 0 \quad (3.8)$$

$$v \frac{\partial u}{\partial r} = \frac{1}{\rho_{nf}} \frac{R_c}{r_c + R_c} \frac{\partial p}{\partial s_c} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial r_c^2} + \frac{1}{r_c + R_c} \frac{\partial u}{\partial r_c} - \frac{u}{(r_c + R_c)^2} \right) - \frac{vu}{r_c + R_c} + \frac{R_c u}{r_c + R_c} \frac{\partial u}{\partial s_c} + \quad (3.9)$$

$$\frac{g}{\rho_{nf}} [(\rho\beta_t)_{nf}(T - T_\infty) + (\rho\beta_c)_{nf}(C - C_\infty)]$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial r_c} + \frac{R_c u}{r_c + R_c} \frac{\partial T}{\partial s_c} = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial r_c^2} + \frac{1}{r_c + R_c} \frac{\partial T}{\partial r_c} \right) \quad (3.10)$$

$$v \frac{\partial C}{\partial r_c} + \frac{R_c u}{r_c + R_c} \frac{\partial C}{\partial s_c} = D_m \left(\frac{\partial^2 C}{\partial r_c^2} + \frac{1}{r_c + R_c} \frac{\partial C}{\partial r_c} \right) \quad (3.11)$$

With border condition

$$u = U_w + \delta^* \frac{\partial u}{\partial r_c} \text{ as } U_w = a s_c, \text{ where } a \text{ is a constant}$$

$$u = a s_c + \delta^* \frac{\partial u}{\partial r_c}, v = 0, T = T_w, C = C_w \quad \text{at } r_c = 0 \quad (3.12)$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \frac{\partial u}{\partial r_c} \rightarrow 0 \quad \text{as } r_c \rightarrow \infty \quad (3.13)$$

u and v are represented the velocity of fluid in the direction of curvilinear axis s_c and r_c respectively, R_c refers to the Radius of curved surface, $(\rho C_p)_{nf}$ refers to the specific heat, D_m refers to the Molecular diffusivity, ρ_{nf} is density of fluid, k_{nf} refers to Thermal conductivity of the fluid, μ_{nf} refers to the dynamic viscosity of the fluid, & δ^* refers to the slip coefficient.

3.3 METHOD

$$\zeta = \frac{\lambda}{2\pi} \left(\frac{x}{x^2 + (y+d)^2} \right) \quad (3.14)$$

Where λ is for the MF strength. As we know that gradient of the magnitude is an important Expression, which is directly proportional to the body force. The magnetic strength is represented by the following equation, where H represents the magnitude of the MF.

$$H = \sqrt{\left(\frac{\partial \zeta}{\partial x} \right)^2 + \left(\frac{\partial \zeta}{\partial y} \right)^2} \quad (3.15)$$

Components of MFH are

$$\frac{\partial H}{\partial x} = -\frac{\lambda}{2\pi} \left(\frac{2x}{(y+d)^4} \right) \quad (3.16)$$

$$\frac{\partial H}{\partial y} = \frac{\lambda}{2\pi} \left(\frac{-2}{(y+d)^3} + \frac{4x^2}{(y+d)^5} \right) \quad (3.17)$$

Temp is expressed in terms of magnetization M, by &ersson & colleagues.

$$M = K^c (T_c - T_f). \quad (3.18)$$

The dimensionless quantities & transformation are taken into account which is considered by (Chen & colleagues.,1988).

$$\Psi(\tau, \eta) = \frac{\mu}{\rho \tau} f(\eta) \quad (3.19)$$

$$\alpha(\tau, \eta) = \frac{T_c - T_f}{T_c - T_w} \theta(\eta) + \tau^2 \phi(\eta), \quad (3.20)$$

$\omega(\tau, \eta)$ denotes the dimensionless steam Expression, while $\alpha(\tau, \eta)$ denotes temp. This is also how the dimensionless coordinates τ & η are shown.

$$\tau = \sqrt{\frac{c_p}{\mu}} x, \eta = \sqrt{\frac{c_p}{\mu}} y, \quad (3.21)$$

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$\psi(\tau, \eta)$ denotes the dimensionless stream function, while $\theta(\tau, \eta)$ denotes temp. This is also how the dimensionless coordinates τ & η are shown.

$$v = -\frac{\partial \psi}{\partial x} = (cv)^{\frac{1}{2}} f(\eta) \quad (3.22)$$

Substituting Equations. (3.19) - (3.22) into Equations (3.14) - (3.15) & comparing coefficients up to n^2 .

We obtain the following simplified NODEs:

$$f''' + ff'' - f'^2 + \frac{2\beta\theta}{(\eta+\delta_1)^4} - N[2f'f'' - ff'' + (f'')^2] = 0 \quad (3.23)$$

$$\theta'' + \text{Pr}(f\theta' - 2f'\theta) + \frac{2N\beta(\theta-\omega)f}{(\eta+\delta_1)^3} - 4N(f')^2 + 2(\omega^2 - 1) = 0 \quad (3.24)$$

$$\phi'' + \frac{2N\beta f\theta^2}{(\eta+\delta_1)^3} - \text{Pr}(4f'\phi' - f\phi) - N\beta(\theta - \omega) \left[\frac{4f}{(\eta+\delta_1)^3} + \frac{2f'}{(\eta+\delta_1)^4} \right] - N(f'')^2 \quad (3.25)$$

Also border conditions (3.16) & (3.17) are converted as

$$f = S; f' = 1; \theta = 1 + \alpha f''(0); \phi = 0 \quad \text{at} \quad \eta = 0, \quad (3.26)$$

$$f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{at} \quad \eta \rightarrow \infty \quad (3.27)$$

Ferromagnetic interaction parameter $(\beta) = \frac{\lambda \phi K^c}{2\pi \mu_e^2} \mu_e(T_c - T_w)$, Viscoelastic parameter

$(N) = \frac{c\mu^2}{\rho(T_c - T_w)}$, Prandtl number $(\text{Pr}) = \frac{\mu c_p}{k}$, Dimensionless Curie temperature ratio

$(\omega) = \frac{T_c}{T_c - T_w}$, Dimensionless distance $(\lambda) = \sqrt{\frac{k\rho d^2}{\mu}}$, Viscous dissipation $(Y) = \frac{c\mu^2}{\rho k(T_c - T_w)}$

A definition of the Surface friction factor & the coefficient of the Nusselt number is as follows:

$$C_{fx} = \frac{-2\tau_w}{\rho(cx)^2}, \quad Nu_x = \frac{x}{-k(T_c - T_w)} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (3.28)$$

Where

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} + \delta_1 \left(u \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} \right),$$

Using equations (4.12) to (4.16) the following dimensionless physical quantities achieved

$$C_f Re_x^{1/2} = -2(1 - N^*)f''(0)$$

$$Nu_x = -[(\theta'(0) + \xi^2 \phi'(0))] Re_x^{1/2} \quad \theta'(0)$$

After changing the system of DE & border conditions, one may get the system of nonlinear ordinary DE. By converting them to initial value problems & then using the finite difference approach, we have quantitatively addressed the majority of border value difficulties. An assumed Step size of 0.01 is being used. We choose n (max) = 15 & assume an error tolerance threshold of 105 to ensure that the asymptotic convergence conditions are met. The far-field border criteria were satisfied by repeatedly changing the trial values of $\zeta'''(0)$, $j''(0)$, $\varepsilon'(0)$, & $\phi'(0)$.

The non-dimensional governing equations of the fluid flow problem are obtained using the following similarity transformations:

$$\xi = \left(\frac{a}{v_f}\right)^{1/2} r_c, \quad u = as_c g'(\xi), \quad p = \rho_f (as_c)^2 P(\xi)$$

$$v = -\frac{R_c}{r_c + R_c} \sqrt{av_f} g(\xi), \quad T = T_\infty + (T_w - T_\infty)\theta(\xi)$$

$$C = C_\infty + (C_w - C_\infty)\phi(\xi)$$

The similarity transformation mentioned above fulfils Eq. 3.7, & the simplified equations are:

$$\frac{\rho_{nf}}{\rho_f} \frac{\partial P}{\partial \xi} = \frac{g'^2}{\xi + K} \quad (3.29)$$

$$\begin{aligned} \frac{\rho_f}{\rho_{nf}} \frac{2K}{\xi + K} P = \frac{v_{nf}}{v_f} \left(g''' - \frac{1}{(\xi + K)^2} g' + \frac{1}{(\xi + K)} g'' - \lambda g' \right) - \frac{K}{(\xi + K)} (g')^2 + \frac{K}{(\xi + K)} g g'' + \\ \frac{K}{(\xi + K)^2} g g' + \lambda \theta + K_2 \phi \end{aligned} \quad (3.30)$$

$$\frac{1}{Pr} \frac{(\rho C_p)_f}{(\rho C_p)_{nf}} \left(\theta'' + \frac{1}{(\xi+K)} \theta' \right) + \frac{K}{(\xi+K)} g \theta' = 0 \quad (3.31)$$

$$\phi'' + \frac{K}{(\xi+K)} Sc g \phi' + \frac{\phi'}{(\xi+K)} = 0 \quad (3.32)$$

With the associated border condition of the flow issue:

$$g'(0) = 1 - \delta g''(0), g(0) = 0, \theta(0) = 1, \phi(0) = 1 \quad \text{at } \xi = 0 \quad 3.33$$

$$g' \rightarrow 0, g'' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } \xi \rightarrow \infty \quad 3.34$$

In order to remove the pressure term from equations, we differentiate equation no. (3.21) w.r.t. ξ then using the value of $\partial P / \partial \xi$ from Eq. no. (3.20), the obtained equation is:

$$g^{iv} + \frac{2}{\xi+K} g''' - \frac{1}{(\xi+K)^2} g'' + \frac{1}{(\xi+K)^3} g' - \lambda g' + \frac{v_{nf}}{v_f} \left\{ \frac{K}{(\xi+K)} (g g''' - g' g'') + \frac{K}{(\xi+K)^2} (g g'' - g'^2) - \frac{K}{(\xi+K)^3} g g' \right\} + \lambda \left[\theta' + \frac{\theta}{(\xi+K)} + k_2 \left(\phi' + \frac{\phi}{(\xi+K)} \right) \right] = 0 \quad (3.35)$$

Where $\delta = \lambda \sqrt{\frac{a}{v_f}}$ is slip parameter, $\lambda = \frac{\rho \beta_t (T - T_\infty) / v^2}{(s U_w)^2 / v^2}$ mixed convection parameter, $K = R_c \sqrt{\frac{a}{v_f}}$ curvature parameter, $K_2 = \frac{\beta_c (T - T_\infty)}{\beta_t (C - C_\infty)}$ buoyancy ratio parameter. $Sc = \frac{v_f}{D_m}$ is Schmidt number, $Pr = \frac{v_f (\rho C_p)_{nf}}{k_{nf}}$ is Prandtl number.

The Surface friction factor, local Nusselt number coefficient, & Sherwood number of a curved surface are physical parameters that are defined as:

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, \quad Nu = \frac{s q_w}{k_{nf} (T_w - T_\infty)}, \quad Sh = \frac{s j_w}{D_m (C_w - C_\infty)}$$

Where $\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial r_c} - \frac{u}{r_c + R_c} \right)_{r_c=0}$ is shear stress at wall, $q_w = -k_{nf} \left(\frac{\partial T}{\partial r_c} \right)_{r_c=0}$ is surface heat flux, $j_w = -D_m \left(\frac{\partial C}{\partial r_c} \right)_{r_c=0}$ surface mass flux in the direction of s_c axis.

3.4 RESULTS

The conversion of the differential equation & border condition systems is a first step in acquiring the NODES. Our numerical solutions to most border value issues were efficient because we first transformed them into initial value problems & then used the finite difference method. One might make the assumption that the Step size is 0.01. The asymptotic convergence criteria are fulfilled by selecting η (max) = 15 & taking the error tolerance value into account as 105. This choice is made in order to assure that the conditions are met. Following an iterative process of adjusting the trial values of $\varsigma'''(0)$, $f''(0)$, $\varepsilon'(0)$, and $\phi'(0)$, the far-field border criteria were successfully satisfied.

The conversion of the differential equation & border condition systems is necessary prior to acquiring the NOD equations system. In order to purchase the system, this is necessary. By transforming the border value problems into initial value problems & then using the finite difference approach, we were able to efficiently solve most of the border value problems numerically. Because of this, we were able to resolve the border value issues. One might get the conclusion that the Step size is 0.01, if they so want. In order to satisfy the asymptotic convergence criterion, it is necessary to set η (max) = 15 & to take into consideration the error tolerance value of 105. For the purpose of ensuring that the prerequisites are satisfied, this decision has been taken. It was determined that the far-field border conditions were effectively fulfilled after an iterative process of modifying the trial values of $\varsigma'''(0)$, $f''(0)$, $\varepsilon'(0)$, and $\phi'(0)$.

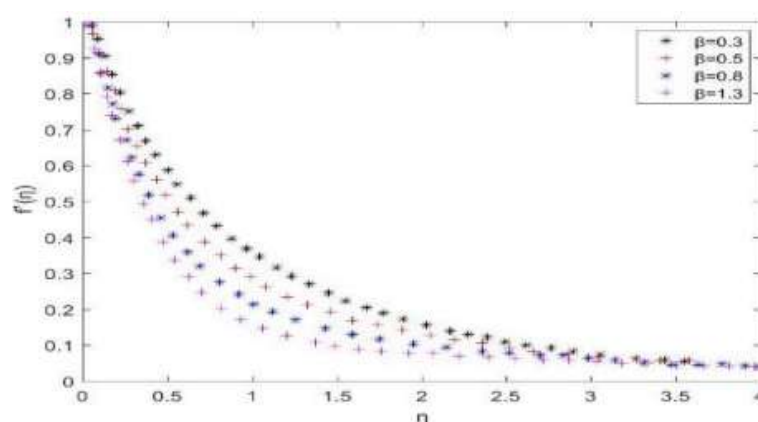


Figure 3.3: $f'(\eta)$ versus β

This figure, 3.3, illustrates the change that takes place for different values of the ferromagnetic interaction parameter β in relation to the velocity profile. When close significant decrease as the value of β grows. This behaviour may be traced back to the presence of ferromagnetic particles inside the ferrofluid. These particles have a strong interaction with the MF that is being applied. Whenever the value of β increases, the magnetic forces that are applied to the fluid become more intense. This, in turn, leads velocity of the fluid. The result of this is that the effective viscosity of the ferrofluid increases, which in turn leads to a decrease in the amount of momentum transfer that takes place inside the border layer. As a consequence of this resistance, the velocity of the fluid falls both in close proximity to the surface and farther away from it. This, in the velocity profile has dropped lends credence to the idea that ferromagnetic contact is the key element that plays a role in defining the behaviour of the flow. When it comes is quite advantageous. Magnetic drug targeting, cooling systems, and ferrofluid-based that has a substantial influence on the hydrodynamic performance of ferrofluid flow.

observation is made, it becomes apparent that the velocity distribution experiences a to the development of a resistive Lorentz force that operates in opposition to the turn, leads to a reduction in the thickness of the momentum barrier layer. The fact that to engineering systems that need flow suppression or stability, the use of such control actuators are a few more examples of systems that fall under this category. The results of the study unambiguously demonstrate that β is a very important regulating parameter

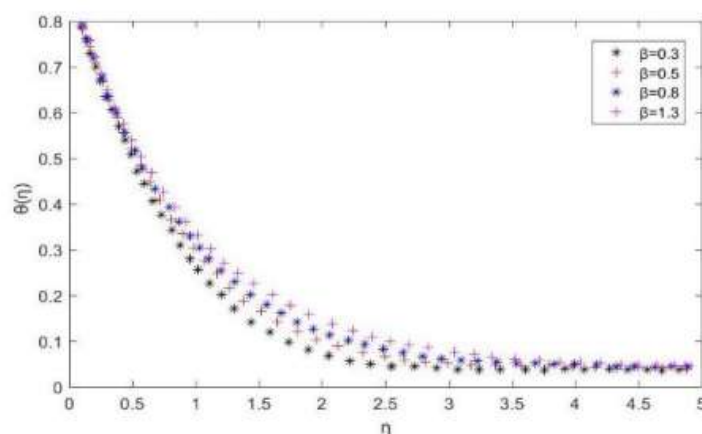


Figure 3.4: (η) versus β

A glance at Figure 3.4 makes it abundantly clear that the velocity of the fluid is a function of the viscoelastic parameter, which is denoted by the letter N . As the value of N increases, the velocity profile continues to increase in quality. Increasing the viscoelastic parameter N results in a decrease in fluid mobility in the vicinity of the stretched sheet, but it results in an increase in fluid mobility farther away from the sheet, as is seen from the graph. When the value of N rises, the transfer of heat is reduced, which means that the fluid may flow more quickly. An increase in N results in an expansion of both the dimensional stream function & the velocity. Figure 3.5 illustrates that when viscoelasticity is present in the fluid, there is a shift in the temp distribution in proportion to the value of n . In proportion to the rise in the viscoelastic parameter, the temp profile will likewise gradually decrease.

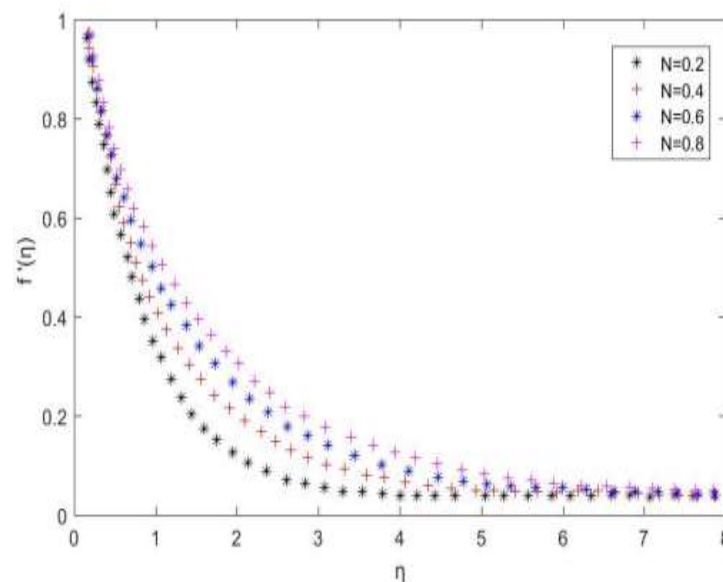


Figure 3.5: $f'(\eta)$ versus N

A representation of the fluctuation of the velocity profile $f'(\eta)$ is shown in Figure 3.5. It is possible for the viscoelastic parameter N to take on a variety of values, which is the source of this variation. Numerous studies have shown that an increase in N results in an increase in the velocity profile across the border layer. It is possible that this activity is connected to the fact that viscoelastic fluids are elastic, which means that they are able to store mechanical energy & then proceed to release it in order to assist in the motion of the fluid. The fluid may now travel with more efficiency because of this. Due to elastic resistance, viscoelastic effects provide a little decrease in mobility in close

contact to the stretched surface. However, the same factors encourage momentum transfer when they are distant from the surface. The overall velocity distribution is improved when N is increased because the fluid exhibits more robust elastic behaviour. This enhancement implies that viscoelasticity contributes positively to flow acceleration, especially at points far from the surface. This sort of behaviour is useful in fields where viscoelastic fluids are regularly encountered, such as biomedical fluids, coating flows, & polymer processing.

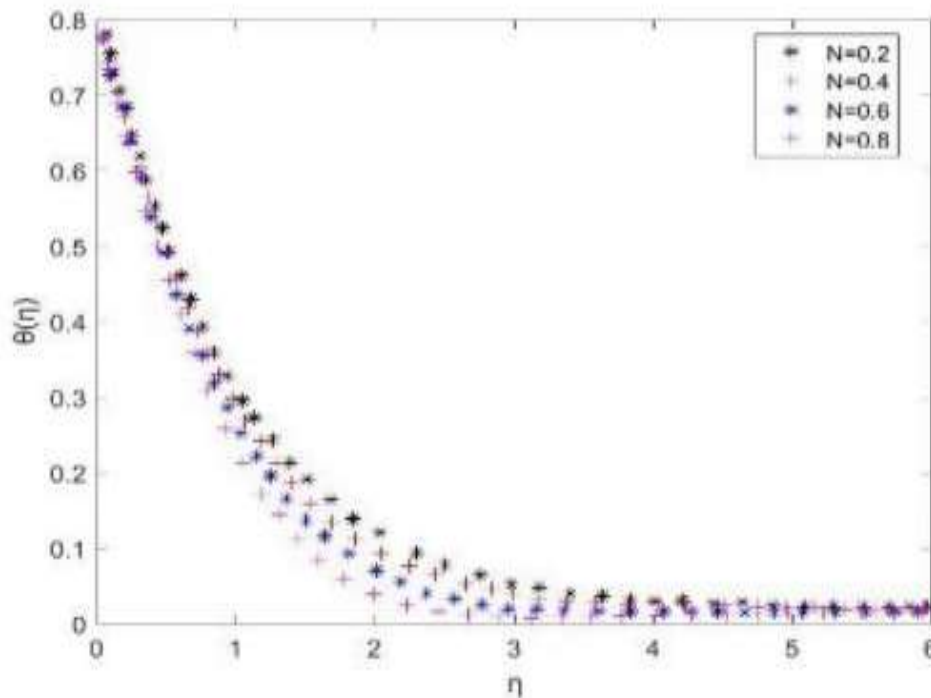


Figure 3.6: (η) versus N

As may be shown in Figure 3.6, the viscoelastic parameter N considerably affects the temp profile $\theta(\eta)$. A decrease in temp is a direct result of the increased nitrogen content. A thinner thermal border surface is the end consequence of viscoelastic processes that reduce the amount of Heat exchange within the fluid, leading to this drop. The fluid's reduced thermal energy retention & enhanced heat loss from the border are both caused by the predominance of the elastic properties. Increasing the viscoelastic parameter values improves heat transmission efficiency, as shown by the decrease in the temp profile. These results suggest that viscoelastic fluids might be

used effectively in situations requiring the control of thermal behaviour. Cooling systems & heat exchangers using polymeric fluids are examples of such uses.

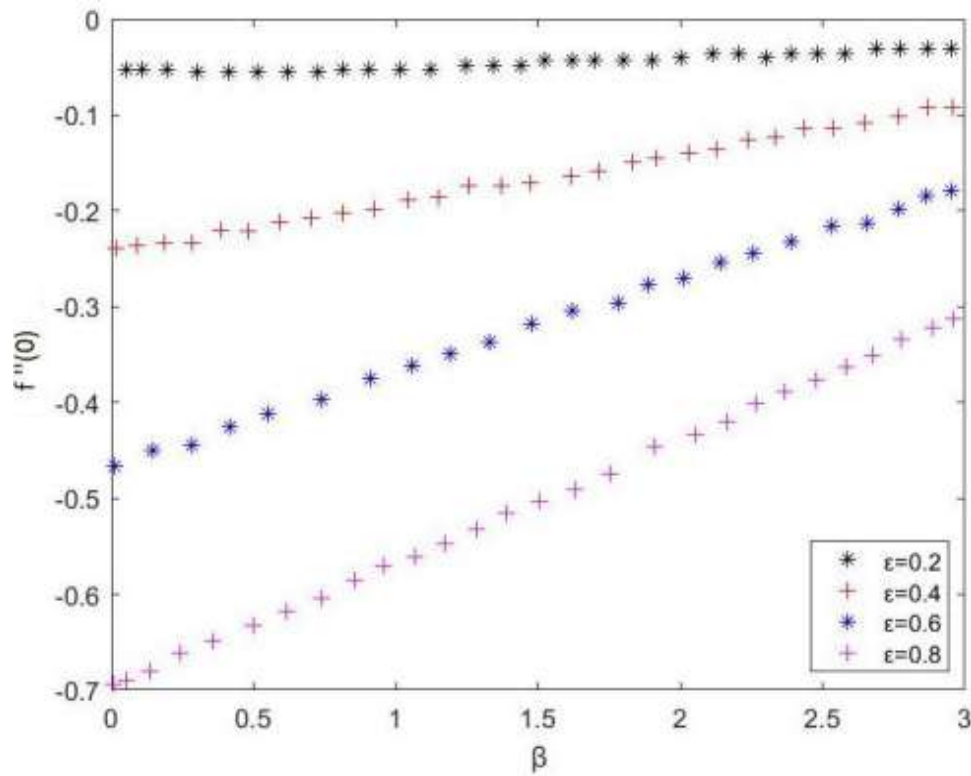


Figure 3.7: Wall shear coefficient versus variable β

Figure 3.7 is a graph that illustrates the variation in the skin friction coefficient as a function of the ratio parameter ϵ . As the size of ϵ increases, there is a discernible decrease in the skin friction coefficient. Since ferrofluid motion is more common than plate motion, the observed behaviour implies that surface shear stress decreases as the ϵ values are increased. Because the fluid velocity adjusts to lessen frictional forces, the resistance at the wall decreases as ϵ rises. Reduced energy loss is one advantage of skin friction minimisation in fluid transport systems. We may deduce that ϵ might be a valuable parameter for managing surface drag in ferrofluid applications based on these findings.

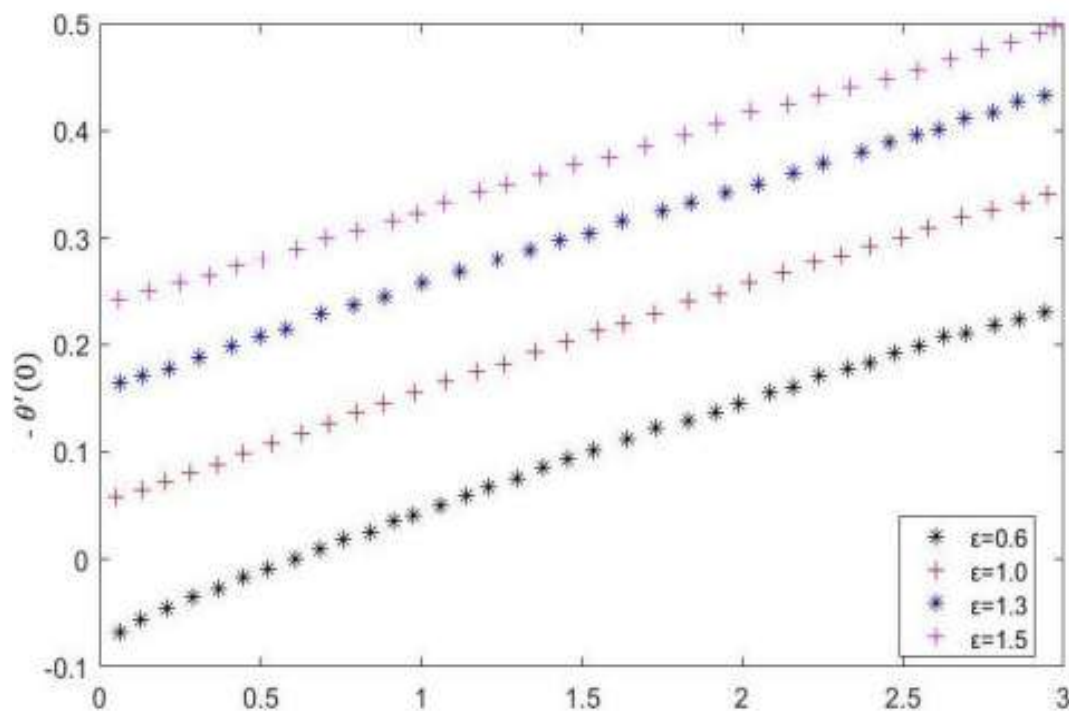


Figure 3.8: Local Nusselt number versus variable E

In Figure 3.8, a visual depiction is provided that illustrates the impact of the ratio parameter ϵ on the Nusselt number representing the local region. As the value of ϵ increases, the local Nusselt number decreases, which results in a decrease in the rate of heat exchange at the surface. This is shown by the measurements. The consequence of this is that the rate of transfer of heat through the surface has slowed down. It is possible that this drop is the result of the events that were discussed before, which result in a thickening of the thermal border layer and, as a consequence, a reduction in the temp differential occurring at the wall. When the value of ϵ is raised, thermal diffusion emerges as the dominant mechanism, resulting in a reduction in the quantity of heat that is transported between the surface and the fluid. These findings have significant repercussions for the development of thermal systems in the future, which are critically essential in a wide variety of applications that involve the transportation of heat.

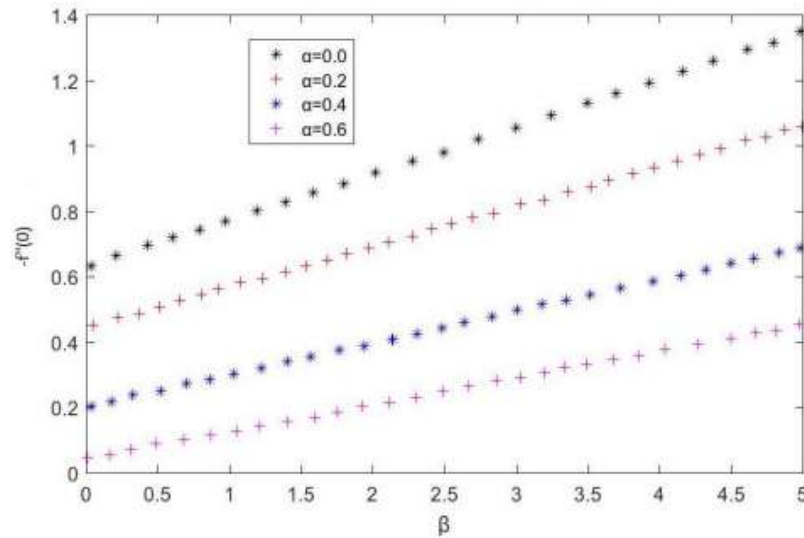


Figure 3.9: Wall shear coefficient versus variable α

Visual representation of the impact of the slip parameter α on the Wall shear coefficient can be seen in Figure 3.9. The coefficient of Surface friction increases as α grows, indicating that surface shear stress increases when slip conditions are present. The velocity gradient near the wall is altered by slip, leading to an increase in the internal momentum exchange &, ultimately, acceleration. Consequently, when sliding is not present ($\alpha = 0$), the Wall shear coefficient is at its minimum. These results highlight the significance of slip effects in flows at the micro- & nano-scale. These flows do not follow the usual no-slip pattern.

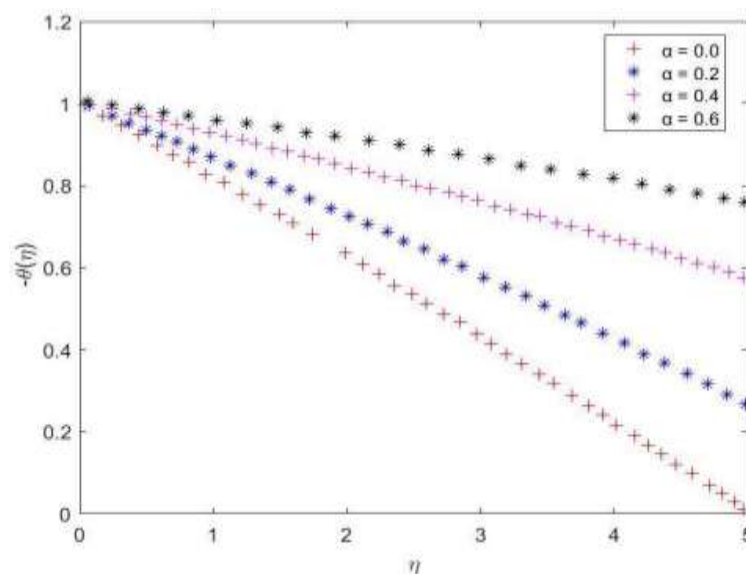


Figure 3.10: Local Nusselt number versus variable α

In Figure 3.10, the variation of the local Nusselt number is shown as a function of the distance between the fluid and the solid. When the slip parameter a is increased, the local Nusselt number experiences a gradual decrease in a monotone manner. According to this pattern, the rate of transfer of heat from the surface to the fluid is in the process of reducing. During the process of slip development, the flow-wall contact reduces, which results in a lower velocity gradient and, thus, a smaller surface temp difference. Because

of the direct connection that exists between the wall temp gradient and the Nusselt slip parameter a , which reflects the amount of velocity slip that occurs at the interface number, decreases in the gradient will result in a reduction in the rate at which heat is transmitted. A further reduction in the surface heat flow occurs when the thickness of the thermal border layer grows as a result of an increase in the value of a . Because of the surface characteristics and rarefaction, many thermal systems at the micro and nano level are susceptible to slip effects. Some examples of these systems are microchannel heat exchangers and microfluidic cooling devices. The significance of this conduct is significantly increased due to the functioning of these systems. According to the results, increasing slip is not a good idea for applications that need quick transfer of heat; nevertheless, it might be advantageous for applications that seek thermal insulation and for applications that want to manage heat exchange.

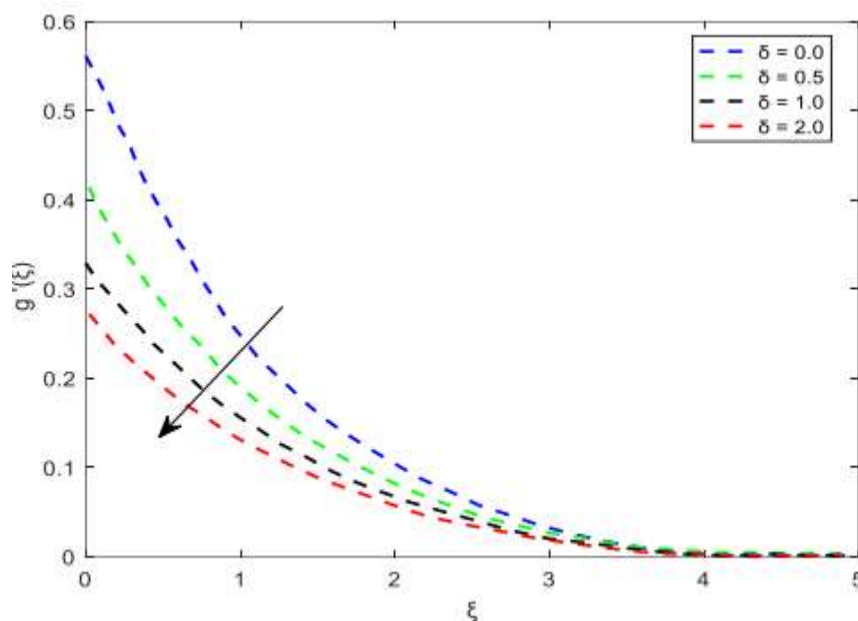


Figure 3.11: $g'(\xi)$ versus variable δ

Figure 3.11 illustrates the impact that the slip parameter δ has on the velocity profile that is present inside the border layer. The graphic provides a clear demonstration that the velocity distribution undergoes a large drop as the slip parameter δ grows. This drop in velocity takes place as a result of greater slip parameters, which effectively raise the dynamic viscosity near the border. This, in turn, increases the barrier to fluid motion. Because of the higher slip, the momentum transfer mechanism between the fluid & the surface is altered, which results in a decreased velocity gradient & a suppression of fluid motion across the border layer. The consequence of this is that the momentum barrier layer grows thinner, & the total velocity of the fluid falls when this occurs. This inverse connection between slip-induced viscosity effects & velocity demonstrates the significant importance that slip circumstances play in influencing flow characteristics, especially in applications that are on a microscale. Because slip effects may be adjusted to alter flow resistance & shear stresses at the surface, the results are significant for the design of microfluidic devices, lubrication systems, & biomedical flows. These are all areas in which slip conditions can be controlled.

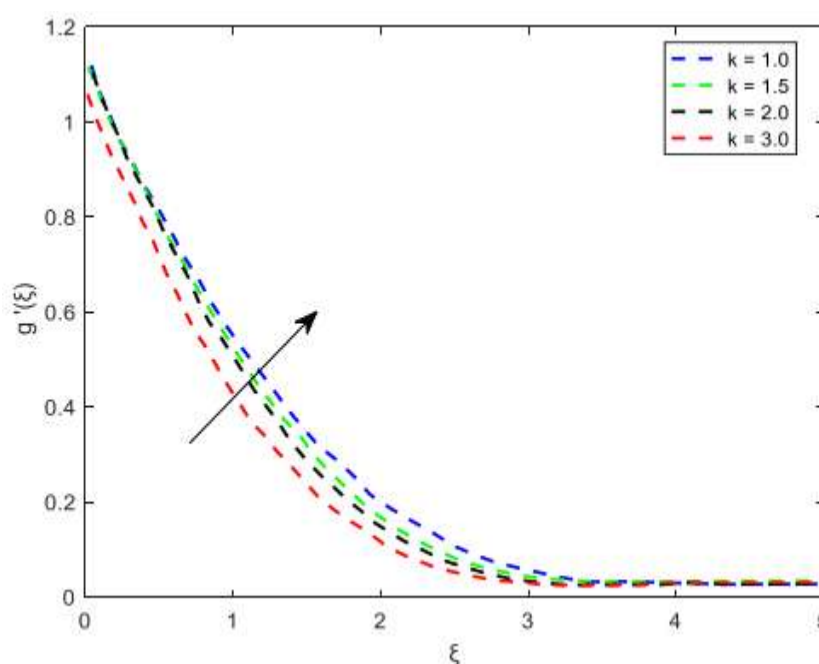


Figure 3.12: $g'(\xi)$ versus variable k

An illustration of the fluctuation of the velocity profile with regard to the curvature parameter K may be seen in Figure 3.12. The velocity profile is shown to grow when the curvature parameter K is increased, as can be seen from the figure. This increase in

velocity may be ascribed to the geometric impact of the curved surface, which makes the motion of the fluid smoother & minimizes the resistance that exists inside the border layer. The expansion of the effective flow area that occurs when the curvature parameter rises makes it possible for the fluid to accelerate over the surface with more freedom. This causes the momentum border layer to become denser, & it also causes the velocity values to increase over the flow domain. The findings not only shed light on the major impact that surface curvature has on fluid dynamics, but they also imply that curved or expanding surfaces might be deliberately employed to improve flow performance. Enhanced fluid motion may lead to increased transport efficiency & lower energy losses in applications that include curved channels, aerodynamic surfaces, & Heat exchange equipment. This behavior is especially helpful in these types of applications because it can lead to better fluid motion.

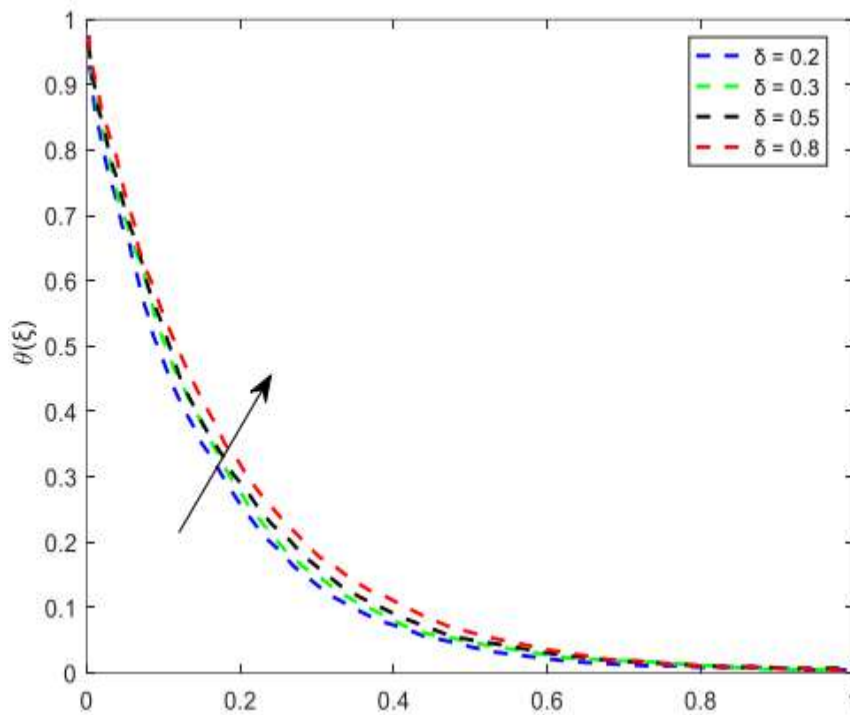


Figure 3.13: (ξ) versus variable N

Figure 3.13 provides a visual illustration of the influence that the slip parameter δ has on the temp profile (π) (ξ) that is present inside the border layer. The increase in the slip parameter δ & the rise in the temp distribution are clearly correlated with one another, leading to an obvious link. One possible explanation for this phenomenon is the weakening of the fluid-surface thermal contact due to velocity slip at the border.

Because the barrier to fluid motion near the wall gets reduced as δ increases, the amount of surface heat carried by convection decreases. When heat builds up within the fluid, it causes the temp across the border layer to rise. The thermal border layer is also becoming thicker, which is a positive development since a smaller wall temp differential means less heat loss. This phenomenon is especially relevant in systems working on the micro- & nano-scales, where slip effects are gaining importance due to rarefaction & surface characteristics. The findings show that raising the slip parameter δ is not a good option when efficient heat evacuation is needed. However, it could work well in places with thermal insulation or where the temp is controlled.

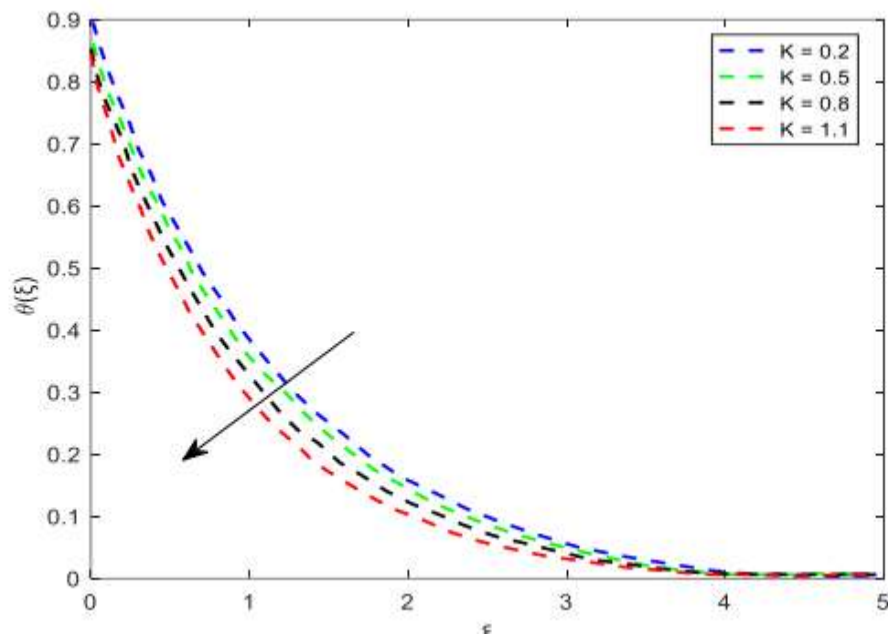


Figure 3.14: $\theta(\xi)$ versus variable ξ

Figure 3.14 shows the data-curvature parameter K relationship, as well as the temp profile variation ($\theta(\xi)$). Increasing the curvature parameter K causes a less uniform distribution of temp, as is seen from the graph. The curvature of the surface increases the velocity of the fluid, which causes more heat to be dissipated away from the surface. The temp drops as a consequence of this. By increasing the effective flow area as K increases, convection increases & the quantity of thermal energy stored within the border layer decreases. A thinner thermal border layer is the end consequence of the geometric effect generated by the curvature, which improves heat transmission from the fluid to the surrounding environment. Surface shape is shown to have a crucial

influence in influencing thermal characteristics by illustrating this phenomenon. Some thermal management devices, such as heat exchangers, cooling fins, & curved channel flows, might enhance Heat exchange by using curved or expanding surfaces. Based on the findings, it seems like these surfaces might be used effectively.

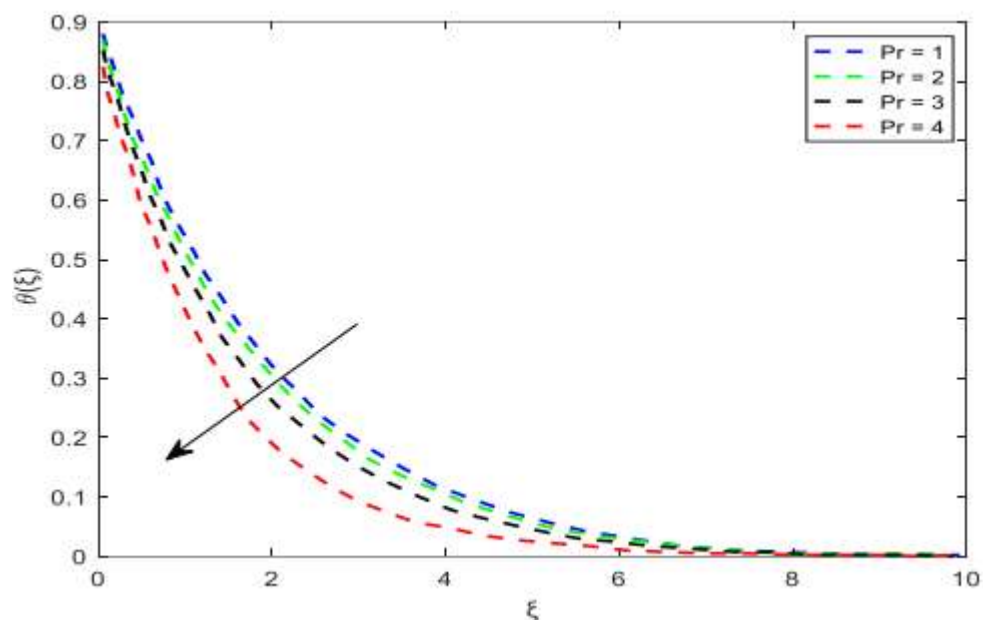


Figure 3.15: $\theta(\xi)$ versus variable P

Take a look at Figure 3.15 to see how the Prandtl number P_f influences the temp profile $\theta(\xi)$ in the context that has been supplied. With the help of the Prandtl number, one can formulate an equation that represents the ratio of the momentum diffusivity to the heat diffusivity factor. It has been discovered that the temp profile experiences a substantial reduction when the value of P_f becomes higher. The decrease in thermal diffusivity that occurs with higher Prandtl numbers is responsible for this phenomenon. This decrease in thermal diffusivity restricts the amount of heat that can be transferred through the fluid. The thermal barrier layer grows thinner as a consequence of this, & the temp inside the fluid drops as a consequence thereof. Higher Prandtl numbers are associated with fluids that have a poor thermal conductivity, such as oils, which have a lower rate of heat diffusion in comparison to momentum diffusion. The steeper temp gradients are responsible for the drop in temp profile, which implies that the surface is experiencing increased Heat exchange rates. These findings are in agreement with the traditional idea of heat transmission & provide more evidence that the Prandtl number constitutes a

significant factor in determining thermal behavior. These discoveries are especially pertinent for industrial operations that include fluids that have varied thermal characteristics.

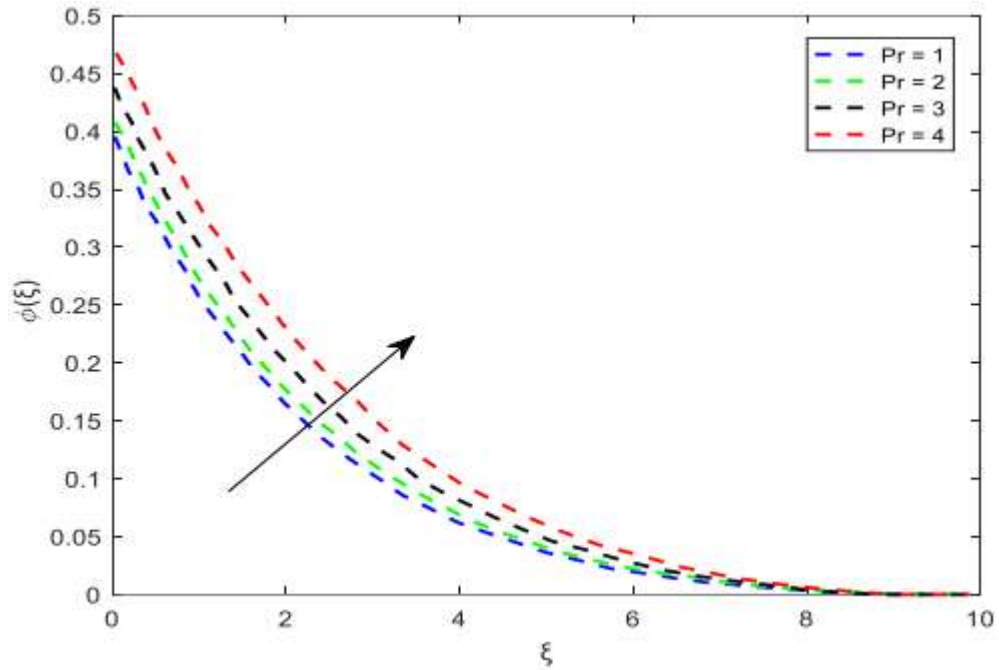


Figure 3.16: $f'(\xi)$ versus variable Pr

Figure 3.16 illustrates the link between the two variables by showing the variation of the velocity profile $f'(\eta)$ with respect to the mixed convection parameter λ . As the value of λ keeps going up, the velocity distribution shows a noticeable increase in magnitude. One probable explanation for this behaviour is the increasing strength of buoyant forces relative to inertial forces in the flow field. The effects of thermal buoyancy become more noticeable as the value of λ increases, which in turn makes it easier for fluids to flow near the surface and increases their velocity. Viscosity gradients are steeper close to the wall and border layer velocities are larger because the buoyant forces are amplified and the fluid movement barrier is lowered. As a result, as the value of λ grows, the momentum barrier layer becomes thicker. In the presence of both forced and natural convection processes, this discovery highlights the important role of mixed convection in improving momentum transmission. Utilising buoyancy-assisted flow may enhance fluid circulation and overall system performance in thermal systems including inclined plates, heat exchangers, and cooling devices. Because of their

potential use in improving fluid circulation, the findings have special relevance for these systems.

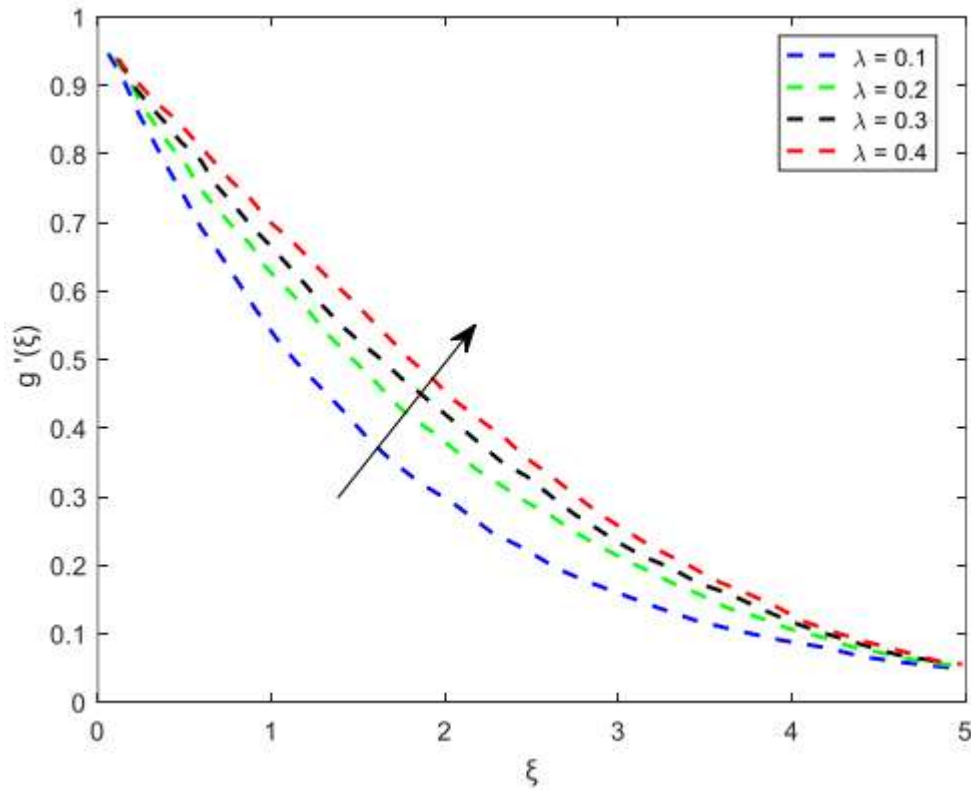


Figure 3.17: $g'(\xi)$ versus variable ξ

The visual representation of the impact of the mixed convection parameter λ on the temp profile $(\eta) \tilde{u}$ may be seen by referring to Figure 3.17. In contrast to the velocity profile, the temp distribution is shown to decrease as the value of η rises with increasing values. This temp drop might be explained by an increase in the rate of convective transfer of heat, which occurs as a consequence of stronger buoyant forces present in the environment. A greater λ value leads to an increase in the fluid's velocity, which in turn improves mixing and facilitates a more efficient transfer of heat energy away from the surface. Due to increased convection, the amount of time hot fluid may stay close to the wall is reduced, leading to lower temp throughout the border layer. Furthermore, when the thickness of the thermal barrier layer decreases, the efficiency of heat transmission increases. As a consequence of these findings, it is clear that mixed convection plays a significant part in the regulation of thermal behavior and that it may be efficiently employed to improve heat dissipation in engineering systems that include buoyancy-driven flows and inclined surfaces.

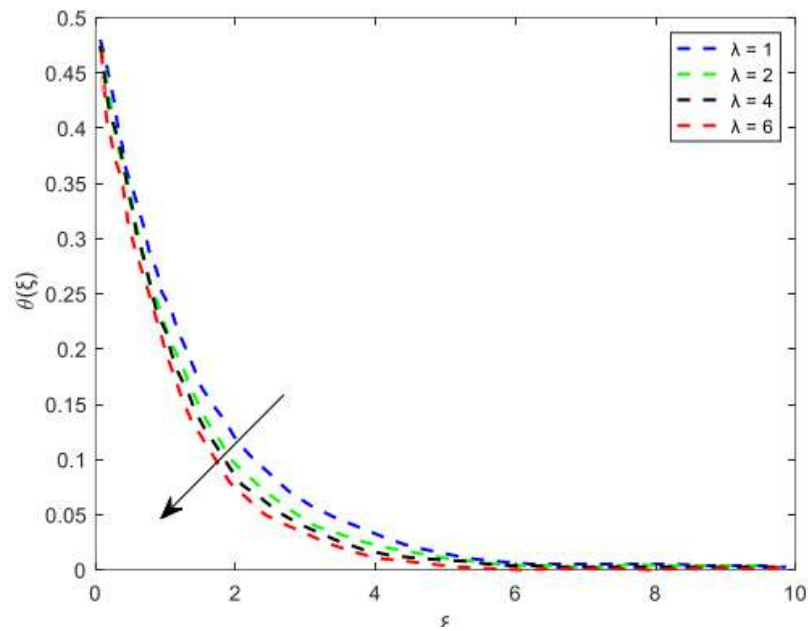


Figure 3.18: (ξ) versus variable p

The Figure 3.18 in showsthis variation is based on the mixed convection parameter λ . By observing the graphic, it becomes apparent that the concentration distribution diminishes as the value of λ grows. This behavior is principally caused by the amplification of buoyant forces, which improves the mobility of fluids & increases the amount of mass that is transported away from the surface environment. The convective currents become stronger as λ increases, leading to better species transport & thinner concentration border layers. The larger pressure gradient associated with increased buoyant forces is another contributor to the drop in concentration levels inside the border layer. Since mixed convection enhances the rates of Transport of species, it is advantageous for applications such as chemical processing, drying technologies, & mass diffusion control, according to this conclusion. The observed patterns show how the Transport of species, Heat exchange, & momentum transfer mechanisms in mixed convection flows are very coupled with each other.

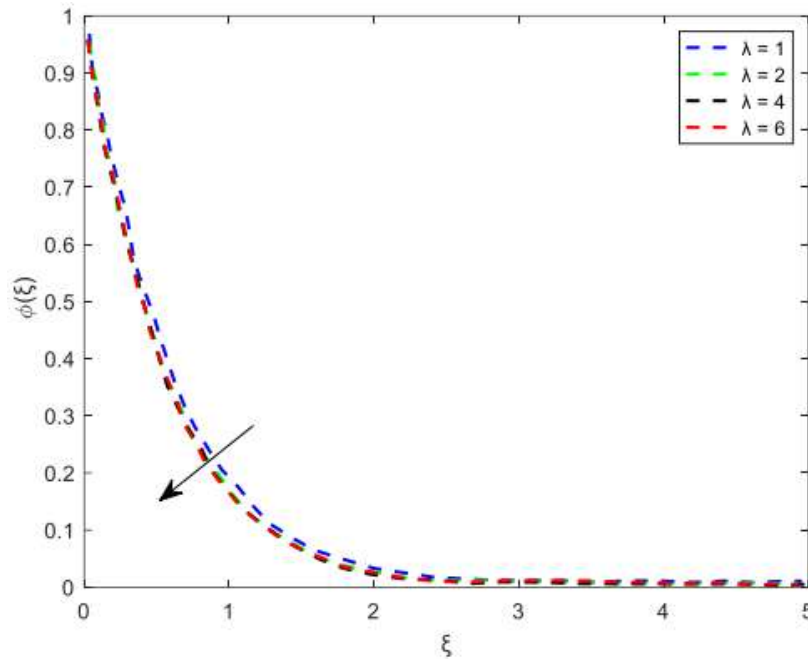


Figure 3.19: (ξ) versus variable n

The Figure 3.19 is shows, which highlights the relationship between the mixed convection parameter and those variations. Upon careful observation, it becomes evident that the concentration distribution gradually diminishes as the value of grows. The increase in buoyant forces, which boost the convective transport of species away from the surface, is responsible for this behavior, which may be related to the intensification of buoyancy forces. The buoyancy-assisted flow becomes more robust as the value of is raised, which leads to a rise in the velocity of the fluid and the mixing that takes place inside the border layer for the fluid. This enhanced convective mobility not only has the effect of reducing the thickness of the concentration border layer, but it also has the effect of accelerating the process of mass diffusion from the surface into the fluid that is around it. Another element that contributes to the drop in concentration levels in the area of the wall is the increase in buoyant forces, which also leads to a more pronounced pressure gradient. This is another aspect that contributes to the decline. The data that have been provided here demonstrate that mixed convection plays an important part in defining the characteristics of mass transfer and may be used effectively in a wide range of processes, such as chemical reactions, drying activities, and mass transport systems.

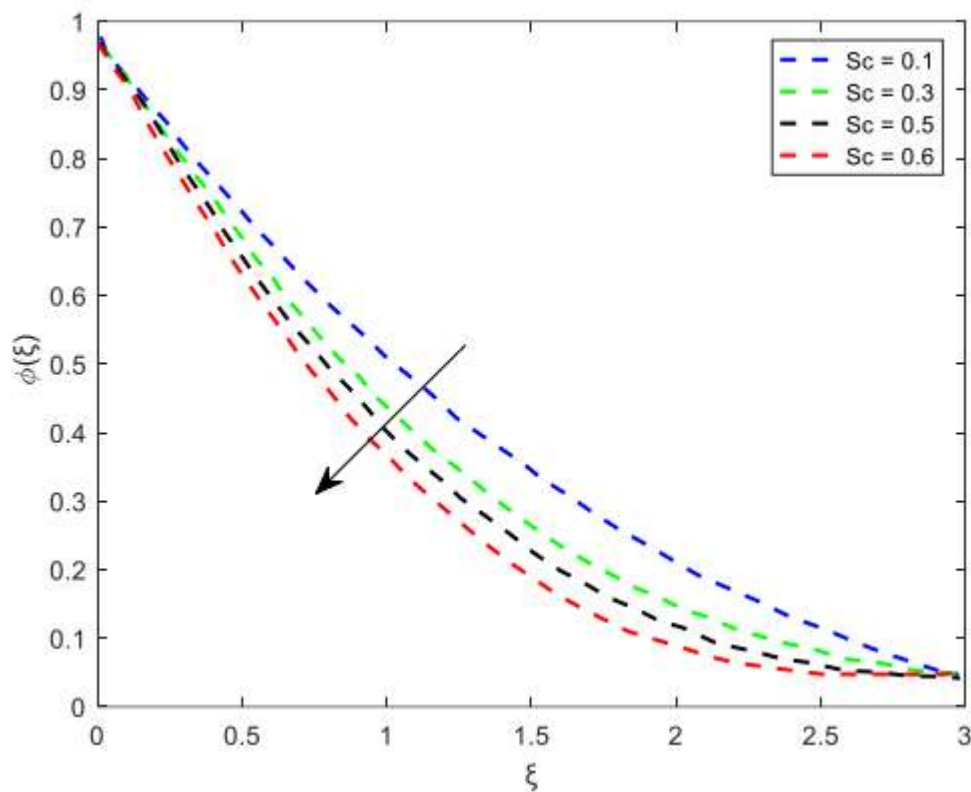


Figure 3.20: (ξ) versus variable S

A graphic example of the influence that the Schmidt number Sc has on the concentration profile (n) may be seen in Figure 3.20. A ratio that illustrates the connection between momentum diffusivity and mass diffusivity is referred to as the Schmidt number. It is seen in the image that the concentration distribution will further decrease as Sc climbs in value. This phenomenon takes place as a result of the fact that greater Schmidt numbers are associated with reduced mass diffusivity, which in turn restricts the movement of species within the fluid. A steeper concentration gradient at the wall and lower concentration values across the flow domain are the results of a rise in Sc , which causes the mass transfer border layer to become thinner. The decrease in the concentration profile is indicative of increased mass transfer rates from the surface. These results are in agreement with the traditional theory of mass transfer and bring to light the significant dependency of concentration distribution on the Schmidt number. The findings are especially pertinent for applications that include the dispersion of species, the processing of chemicals, and the phenomena of environmental movement.

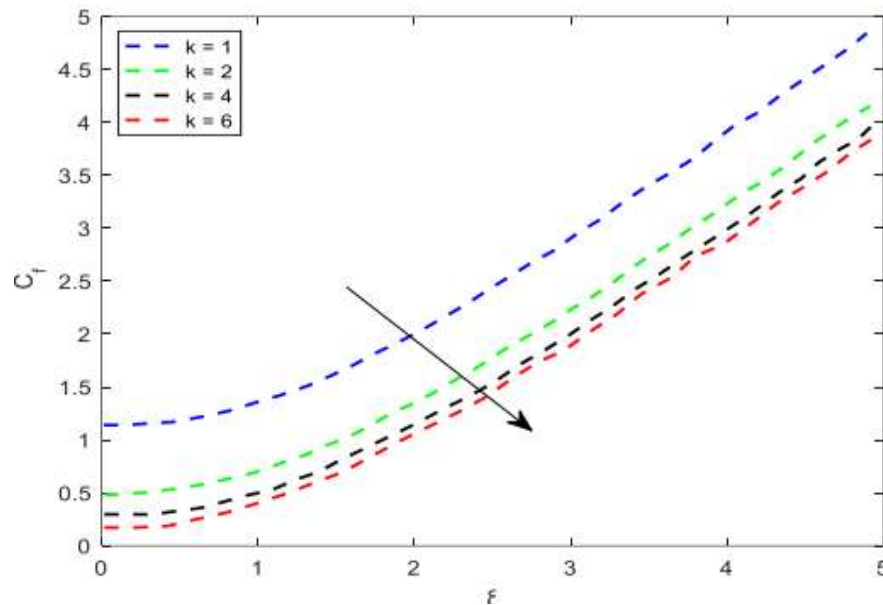


Figure 3.21: Wall shear coefficient versus variable n

Figure 3.21 provides a visual representation of the variation of the Wall shear coefficient in relation to the curvature parameter K . Please refer to this figure for more clarification. The Wall shear coefficient has been observed to decrease as the curvature parameter K is raised. This was discovered via research. This decrease in surface shear stress is brought about by the geometric impact of surface curvature, which makes fluid motion more smooth & decreases the amount of resistance that exists inside the border layer. Increases in the curvature parameter result in an expansion of the effective flow area, which in turn enables the fluid to accelerate while experiencing a reduction in the frictional losses. The reduction in Surface friction is advantageous since it helps to reduce the amount of energy that is lost over time & improves flow efficiency. It is suggested by these findings that curved surfaces may be strategically used to maximize flow performance in engineering applications such as aerodynamic surfaces, curved channels, & Heat exchange equipment. These findings also emphasize the significance of surface geometry in the process of managing drag forces.

Table 3.1: Numerically computed results of $-\theta'(0)$ & $-\phi'(0)$ for the different values of K, Pr, Sc .

K	Pr	Sc	$-\theta'(0)$	et al.	$-\phi'(0)$	et al.
1	1.0	0.6	1.081935728		1.087855437	1.08785273
3	1.3	0.8	1.082142173		1.085473158	
5	1.4	1.0	1.082527168	1.08252	1.140732428	1.14073275
6	1.5	1.2	1.084471589	1.084473678	1.175715642	

Table 3.1 displays the calculated dimensionless heat & Transport of species rates $-\theta'(0)$ & $-\phi'(0)$ for different permeability, Pr & Schmidt numbers. The study found that increasing the permeability parameter K improves fluid penetration in porous media, leading to higher heat & Transport of species rates at the surface. Increased Pr number Pr , linked to reduced thermal diffusivity, enhances heat transmission by thinning the thermal border layer, but has a lesser impact on Transport of species. As the concentration border layer grows thinner, raising the Schmidt number Sc , which indicates less mass diffusivity, leads to larger values of $-\phi'(0)$, indicating more Transport of species at the surface. The table shows how permeability, thermal diffusivity, & mass diffusivity factors affect surface heat & mass transmission characteristics.

3.5 CONCLUSION

A comprehensive numerical & theoretical examination of a two-dimensional stagnation point ferrofluid flow that is subject to velocity slip border conditions has been described in this chapter. The need of comprehending the combined impacts of magnetic interaction, slip mechanisms, viscoelastic behavior, & thermal transport in border layer flows was the impetus for the research that was carried out. The study captures an essential type of fluid motion that is often found in industrial processes, cooling systems, & magnetic fluid applications. This is accomplished by concentrating on stagnation point flow.

The appropriate physical assumptions were taken into account throughout the process of generating the partial DE that regulates the processes of momentum transfer, energy transfer, & concentration transfer. By using appropriate similarity variables, these equations were converted into a system of linked nonlinear ordinary DE. This transformation had been accomplished. The mathematical side of the problem was greatly reduced as a result of this adjustment, but at the same time the fundamental physics of the flow behaviour at the border layer was maintained.

Because of the extremely nonlinear character of the equations that were produced, it was not possible to arrive at analytical solutions. It was necessary to use severe numerical approaches in order to get correct findings for the concentration,

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temperature, and velocity profiles. It was possible to carry out a comprehensive parametric analysis of the physical characteristics that were accountable for this investigation because of the efficient and trustworthy numerical technique that was utilised in this investigation.

The analysis of mixed convective nanofluid flow in the presence of ferromagnetic interactions and velocity slip effects was the major emphasis of the study that was being presented here. A special emphasis was made on acquiring an understanding of the manner in which a variety of physical parameters influence the fluid concentration, temperature behaviour, and momentum transfer that occur inside the border layer. Through the use of first- and second-order velocity slip factors, it was feasible to get a more precise picture of the flow behaviour in close proximity to the surface.

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A thorough investigation of the ferromagnetic interaction variable was carried out in order to get an all-encompassing comprehension of flow characteristics. It has been shown via numerical analysis that whenever there is an increase in the ferromagnetic interaction variable, there is a corresponding decrease in the velocity profile. When the magnetic force that acts as a resistance causes the velocity profile of the fluid to decrease, this phenomenon takes place. It is a phenomenon that occurs only when the

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magnetic force is present. On the other h&, the temperature profile shows a large rise if the values of the ferromagnetic properties are raised to higher standards. There is a possibility that this phenomenon might be explained by the fact that magnetic interactions result in a larger destruction of energy.

Furthermore, the effects of the viscoelastic parameter on the flow & temperature fields were investigated. This was done in addition to the previous point. Increasing the viscoelastic value leads to an improvement in the velocity profile, which shows that the amount of momentum carried by the fluid also improves as the viscoelastic value grows. A higher viscoelastic value enhances the velocity profile. The temperature field becomes more constrained as a result of an increase in this parameter, which further shows that viscoelastic processes play a substantial role in determining the parameters of heat transmission.

In connection to the ratio parameter, there were a great deal of additional engineering elements that were investigated. The skin friction coefficient and the local Nusselt number were two of the important components that were considered. Increasing the ratio parameter leads to a drop in both the skin friction coefficient and the local Nusselt number, according to the data. This trend is seen when the ratio parameter is increased. As a result of the existence of this pattern, it is easy to see how surface shear stress and the rate of heat transfer are sensitive to variations in flow parameters.

There was a discernible relationship between the velocity slip parameters and the surface characteristics. The local Nusselt number decreases as the skin friction coefficient increases with increasing slip parameters. That slip circumstances enhance shear effects and diminish surface heat transmission is a reasonable inference to draw. This phenomenon plays a pivotal role in flow applications at the micro- and nano-scales.

A downward trend in the velocity field is brought about by increasing the value of the second-order slip parameter. Another option is to increase the curvature parameter, which will result in a steeper velocity profile. Surface shape & slip mechanisms are responsible for controlling the transfer of momentum inside the border layer. This seemingly paradoxical behaviour is an illustration of the profound interplay that exists In relation to these two factors.

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A downhill trend may be seen in the velocity field if the value of the second-order slip parameter is increased. This trend is characterised by a downward tendency. On the other hand, the curvature parameter has an opposite consequence, which is to increase the velocity profile. On the contrary, this results in the opposite consequence. The following illustration illustrates the complex interplay in relation to surface geometry & slip processes, which is responsible for determining the momentum transfer inside the border layer. This interaction is difficult to understand because of the contradictory behaviour that occurs. The border layer moment is determined by this interaction, which is accountable for its definition.

Concerning the Prandtl number, an investigation was carried out to examine the concentration distribution that existed inside the border layer. As the Prandtl number grows, the numerical findings demonstrate that the concentration profile also increases, which indicates that the mass diffusion effects are becoming more pronounced. In contrast, the temp profile experiences a reduction as the Prandtl number increases, which is a reflection of the decreased thermal diffusivity that occurs at higher Prandtl values.

The effect of the mixed convection parameter on the flow patterns was also the subject of research. Research has shown that raising the mixed convection value leads to an improved velocity profile. This demonstrates the critical role of buoyant forces in enhancing the velocity of fluids near the stagnation zone.

Increases in the curvature parameter were shown to decrease the Wall shear coefficient, according to the study. Potentially useful in engineering design are fluid flow systems that realistically use surface curvature to reduce frictional resistance. Surface curvature may be used, as seen by this behaviour.

Ultimately, this chapter aims to precisely numerically analyse ferrofluid flow near the stagnation point when velocity slip is present. Through the use of velocity, temp, concentration, & other crucial engineering factors, the combined effects of ferromagnetic contact, viscoelastic behaviour, slip parameters, curvature, & mixed convection have been made evident. A number of engineering quantities were used to achieve this. The provided tables & figures aim to clearly show how governing factors affect thermal & fluid systems that use ferrofluids, which helps with their optimization & design.

NUMERICAL STUDY OF MHD FLOW WITH VELOCITY SLIP OVER AN INCLINED POROUS PLATE

4.1 INTRODUCTION

Recent times have witnessed an increase in the amount of attention that has been paid to the research of nanofluid flow over porous material that is magnetically hydrodynamically free convection. In the context of industrial settings, the use of MHD free convection is rather frequent. It has applications in a broad range of industries, such as chemical engineering, electrical devices, the manufacturing of polymers, and nuclear power plants, among others. Due to the fact that nanoparticles improve the thermal conductivity of the fluid, nanofluids have been a game-changer in the research of border layer issues. This is because nanofluids have an increased thermal conductivity. The prior work done by Sakiadis and colleagues. (1961) establishes a characteristic of fluid flow difficulties on a flat surface. This study expands on that work. Alam and colleagues. (2007) demonstrated that the MF effect and the convective transfer of heat condition were both present at the same time. No-slip border conditions in two-dimensional nanofluid flow were the subject of research conducted by Ramya and colleagues. (2018). An example of a porous media effect is the MHD flow that is produced by an angled porous plate, which was described by Sulochana and colleagues. (2018). In the study conducted by Mondell and colleagues. (2018), the symmetry analysis method was used to evaluate the flow of nanofluids and the transformation of heat in a porous medium that was placed over an inclined plate.

The flow of an unstable viscous fluid over an inclined sheet is comprehensively investigated in connection to thermophoresis & Brownian motion, as stated in the research conducted by Rasool& colleagues. in the year 2020. An investigation on the impacts of heat production & absorption on heat transmission in two-dimensional multi-mode high-velocity nanofluid flows was carried out by Maleki& colleagues. (2019). There is a symmetric solution to the incompressible MHD flow issue that was

identified by Bhuvaneswari & colleagues (2014). This solution takes into account the chemical features of the fluid. In a research that was published in 2018, Srinivasacharya & colleagues provided a comprehensive analysis of the temp distribution that occurred while fluids flowed freely over an inclined plate. In their 2017 study, Reddy & colleagues looked at the phenomenon of Heat exchange in mixed convective nanofluid flows that were induced by porous medium. Sharma & colleagues. (2001) provided evidence that indicated the influence that porosity has on the flow of fluid in a horizontal channel. M&al & colleagues. (2013) conducted research to determine how the buoyant force influences the flow of multi-homogeneous nanofluids through a porous medium. Micropolar nanofluid flow, thermophoretic motion, Brownian motion, & the influence of viscosity change on the distribution of fluid velocity were all investigated by Rafique & colleagues. (2020).

In accordance with the findings of Srinivas Acharya & colleagues. (2015), the investigation of heat generation & absorption is also carried out within the context of heat transmission across a stretched sheet as a mixed convective fluid flow. In order to provide further information on the micro-channel flow pattern, Fanaee & colleagues. (2021) made use of the most advanced numerical tools currently available. According to Ahmed & colleagues. (2018), the properties of self-convecting nanofluid flows as well as the influence of source & sink parameters on fluid flows in the border layer were discovered. Chamkha & colleagues. (2001) provided a unique numerical technique to fluid modelling that included injection & suction effects at the border layer of the flow. This approach was offered in order to incorporate these effects. In the year 2020, Kushik & colleagues published a report on their research on the flow of ferromagnetic fluids near their point of stagnation in a MF. An approach that takes into consideration both the flow of fluids & the effect that sheet stretching has on the thickness of sheets was discovered by Mishra & colleagues. (2017) via the use of numerical research. In the context of nanofluid flow Heat exchanges, a significant amount of research was conducted on the impacts of thermophoresis & Brownian motion (Kaushik & colleagues., 2021). According to Swain & colleagues.'s research from 2020, the homotopy approach, which incorporates the joule heating condition,

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was successful in resolving the MHD nanofluid flow problem. There is a relationship In relation to velocity slip & the flow of a two-phase fluid when the channel is not uniform. In their study from 2020, Gbadeyan& colleagues explored how the flow of free-stream fluids influences the process of Heat exchange in porous media.

Ahmad and colleagues. (2021) conducted an analysis of the two competing fluid flow models that take into consideration the influence of injection and suction parameters with regard to fluid flow. In their study, Zafar and colleagues. (2021) investigated the transfer of heat that occurs when a chemical reaction is combined with the flow of nanofluid over an inclined plate. (Qiang and colleagues., 2020) investigated the potential impact of the dufour effect on fluid flow in the context of a chemical reaction. Throughout the whole of our investigation, we contrasted the results obtained from computational and analytical methods. In a study that was conducted in 2021, Swaranalathamma and his colleagues attempted to get an understanding of the influence that the first-order velocity slip condition had on transfer of heat. Moreover, the research took into consideration the influence of porous material that was present. Within the scope of their research, Shah and colleagues. (2021) investigated the impact that the velocity slip condition had on the upper-convicted Maxwell fluid. Throughout the whole of the experiment, consideration was also given to the impacts that may be caused by the porous material. The researchers Ramya and colleagues. (2016) discovered that the influence of chemical processes across a stretched, non-linear sheet creates a barrier to the flow of nanofluid inside the sheet. An example of this would be the Newtonian model proposed by Imran and colleagues. in 2020, which is capable of dealing with both homogeneous and heterogeneous characteristics.

The study that was conducted by Ahmad & colleagues. (2021) placed a strong emphasis on the significant role that Brownian motion & the thermophoresis effect play in the transformation of heat & mass that is brought about by an inclined plate. The magnetohydrodynamics& flow over an inclined plate in a porous material under the influence of a convective border layer are the primary topics covered in this chapter. It is also taken into consideration how the speed of the item & the qualities that generate heat would affect the outcome. For the purpose of gaining an underst&ing

of the Heat exchange phenomenon, it is necessary to take into consideration the impacts of thermal slip & velocity slip on the temp& velocity profiles. An important representation of the fundamental system may be achieved by the formulation of pertinent partial DE. For the purpose of solving the nonlinear ordinary differential equation, we make adjustments to the border conditions. The numerical solution of the reduced ordinary DE may be achieved by the use of the finite difference technique. The results have been shown to be significantly influenced by a large number of factors that do not have any dimensions. Some examples of such qualities are Pr , heat production, magnetic, Gr , & velocity slip. Other examples include Pr & heat production. One is possible to demonstrate how significant physical factors influence the velocity & temp profile by using visualisation tools such as graphs & tables. Several pieces of evidence from the data demonstrated that magnetic & permeability properties had a tendency to decrease the velocity profile.

4.2 MATHEMATICAL FORMULATION

We considered a laminar flow over an inclined plate in the present study. 2D flow in the presence of an electrically conducting fluid combined with porous media condition. The angle α indicates that the plate is inclined with respect to the vertical. The x -coordinate is thought of as the direction the plate is facing, whereas the y -coordinate is thought of as being perpendicular to the plate. A evenly distributed MF along the y -axis, with an intensity that depends on the fluid's velocity $B = B_0/Nx$. Where B_0 is constant. T_w is surface temp. T_∞ is ambient temp. The value of T_w is considered greater than T_∞ .

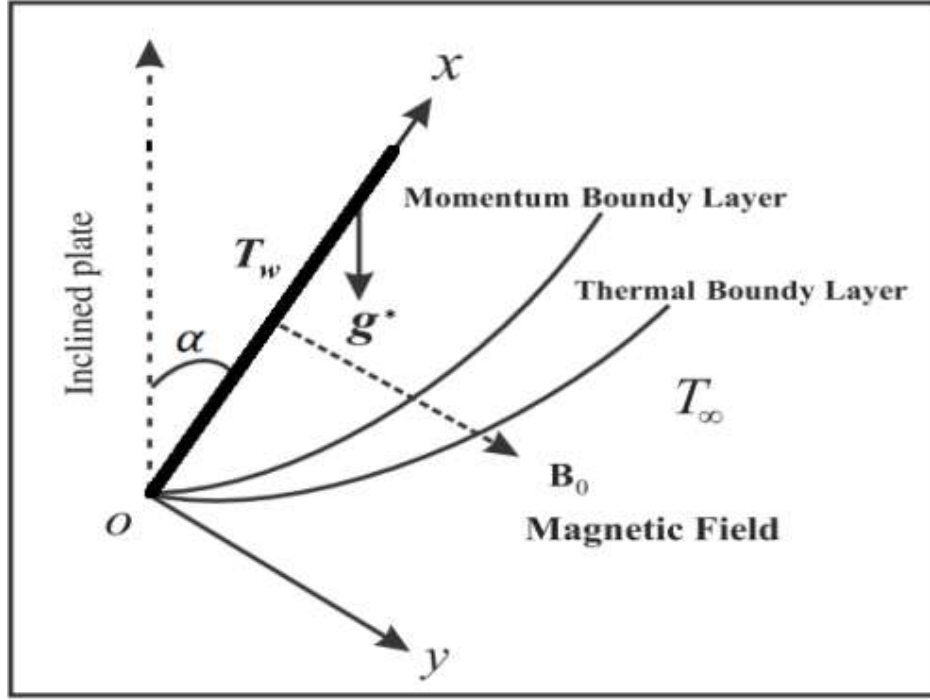


Figure 4.1: Physical model & coordinates system of the flow

With the right border conditions, the flow problem's governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B^2}{\rho} (u - U_\infty) + g^* \beta_t (T - T_\infty) \cos \alpha + \frac{\nu}{k_1} (u - U_\infty) \quad (4.2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{Q_0}{\rho c_p} (T - T_\infty) + \sigma \frac{B^2}{\rho c_p} u^2 \quad (4.3)$$

Associated border conditions are:

$$u = N \nu_f \frac{\partial u}{\partial y}, T = T_w + D \frac{\partial T}{\partial y}, v = 0 \quad \text{at } y = 0 \quad (4.4)$$

$$u = U_\infty, T = T_\infty \quad \text{as } y \rightarrow \infty \quad (4.5)$$

Specifically, M stands for the x- and y-coordinate components of velocity, respectively, in this setting. To indicate the temp of the fluid, the symbol C is used. The velocity slip factor, denoted as N, is equal to $r1x$ minus $n-1/2$, whereas the thermal slip factor, denoted as D, is equal to $D1x$ divided by $-n-1/2$. Kinematic viscosity is represented by

G. Both SZ and (x + b) have the same absolute value. The following symbols are defined in this context: n (free stream velocity), U (constant), Too (free stream temp), k Π (gravitational acceleration), βb (coefficient of thermal expansion), μ (dynamic coefficient of viscosity), a (angle of inclination), and p (fluid density). In scientific notation, the words dk and represent specific heat, thermal conductivity and Q0, respectively, while fluid electrical conductivity and Q0 are employed.

4.3 METHOD

$$\eta = y \sqrt{\frac{U_{\infty}}{2\nu x}}, \quad \xi = \sqrt{2\nu x U_{\infty}} f(\eta), \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}} \quad (4.6)$$

Within this particular context, the variable η is used to represent the similarity variable, (η) is used to represent the non-dimensional stream Expression, & $\theta(\eta)$ is used to represent the non-dimensional tempExpression.

This is achieved by including the similarity transformation into equations (4.1) & (4.3), which ultimately leads to the formulation of equations (4.4) & (4.5).

The resulting equations are:

$$f''' + ff'' - Gr_t \theta \cos \alpha + \frac{2n}{n+1} \omega^2 f' + 2M(\omega - f') - Kf' = 0 \quad (4.7)$$

$$\theta'' + Pr f \theta' + Ec Pr (f'^2 + M f'^2) + Pr Q \theta = 0 \quad (4.8)$$

$w = Uf / U_{\infty}$ represents the velocity parameter, $Gr_t = g \beta (T_w - T_{\infty}) 2x / U_{\infty}^2$ represents the local temp grash of number, n represents the velocity power index, $M = \sigma B_0^2 2x / \rho \nu_{\infty}$ represents the magnetic parameter, and $K = \nu / k_1$ represents the permeability parameter. Ec is equal to $U_{\infty}^2 c (T_w - T_{\infty})$, which represents the Eckert number. Pr is equal to $\nu c p / k$, which represents the Prandtl number. Q is equal to $Q_0 2x / \rho c p U_{\infty}$, which is the local heat production parameter.

With border conditions:

$$f'(0) = 1 + \delta_1 f''(0), \quad \theta(0) = 1 + \delta_2 \theta'(0), \quad f(0) = 0, \quad \text{at } \eta = 0 \quad (4.9)$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (4.10)$$

$\delta_1 = N_1 \sqrt{a(n+1)/2}$ st& for velocity slip parameter, $\delta_2 = D_1 \sqrt{a(n+1)/2\nu}$ st& for thermal slip parameter.

A measure of the skin's amount of friction

$$C_f = \frac{\tau_w}{\rho U_\infty^2}, \text{ where } \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (4.11)$$

The Nusselt number of the local area, defined by

$$Nu_x = \frac{x q_w}{k(T_w - T_\infty)} \text{ where } q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (4.12)$$

$$C_f Re_x^{1/2} = f''(0) \quad (4.13)$$

$$Nu_x Re_x^{-1/2} = -\frac{1}{2} \theta'(0) \quad (4.14)$$

Where $Re_x = U_\infty x / \nu$ is the Local Reynolds number.

In order to solve the requisite nonlinear equation (NDE) with the appropriate border conditions, it is essential to transform the BVP into a system of first-order information value Expressions (IVP). In order to resolve the problem, it is necessary to take some action. For the purpose of providing a precise solution to the DE (4.7) - (4.8) with border conditions (4.9) - (4.10), the Runge-Kutta technique of the fourth order was used.

$$f' = p, \quad \theta' = s, \quad \theta'' = s' \quad (4.15)$$

$$f'' = p' = q$$

$$q' = -fq + Gr_t \theta \cos \alpha - \frac{2n}{n+1} \omega^2 p - 2M(\omega - p) + Kp \quad (4.16)$$

$$s' = -Prfs - EcPr(q^2 + Mp^2) - PrQ\theta \quad (4.17)$$

Under the border condition

$$p(0) = 1 + \delta_1 q(0), \theta(0) = 1 + \delta_2 s(0), f(0) = 0. \text{ at } \eta = 0 \quad (4.18)$$

$$p(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (4.19)$$

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In order to carry out the iterative approach, we make the required beginning assumptions. We have chosen to use $\eta_{max} = 15$ or $\eta_{\infty} = 15$ in order to be in compliance with the border restrictions at infinity throughout the calculation. For the sake of precision, the calculation used a step size of 0.01.

4.4 RESULTS

A numerical analysis was performed on the current flow problem in order to investigate the behaviour of the fluid's transfer of heat rate, velocity, and temp fields. Determining the impacts of the system's thermal performance and flow characteristics as a function of the governing parameters. This is the primary purpose of the research. In order to determine the influence that the following parameters have on the velocity and temp profiles, a comprehensive analysis is carried out. These parameters include the magnetic parameter, the permeability parameter, the thermal Grash of number, the Eckert number, the heat source parameter, the Prandtl number, and the slip parameter. The numerical data that were obtained make it abundantly evident that there is a complex link between these elements and the dynamics of fluid flow.

In order to provide a clear illustration of the impact that each physical parameter has, the numerical solutions are provided in the form of graphs & tables. Unless otherwise specified, the calculations are carried out using the fixed parameter values $Gr_t = 6$, $n = -0.65$, $\alpha = \pi/6$, $Pr = 0.7$, $Ec = 0.2$, $\delta_1 = 0.1$, $K = 1$, $\delta_2 = 0.1$, & ω in the range of 0.5. Throughout the whole of the study, these values are considered to be standard reference parameters. This allows for a consistent comparison of the impacts that are caused by modifications in individual parameters.

This section will provide an explanation of how the velocity & temp distributions of the border layer are affected by the regulating elements that are discussed. When differences in the relevant components are taken into consideration independently, the figures & tables that are linked to this topic give a clear physical comprehension of the patterns that have been noticed. By using this systematic approach, it is possible to get a better understanding of the flow behaviour & Heat exchange characteristics that are associated with the existing mathematical model.

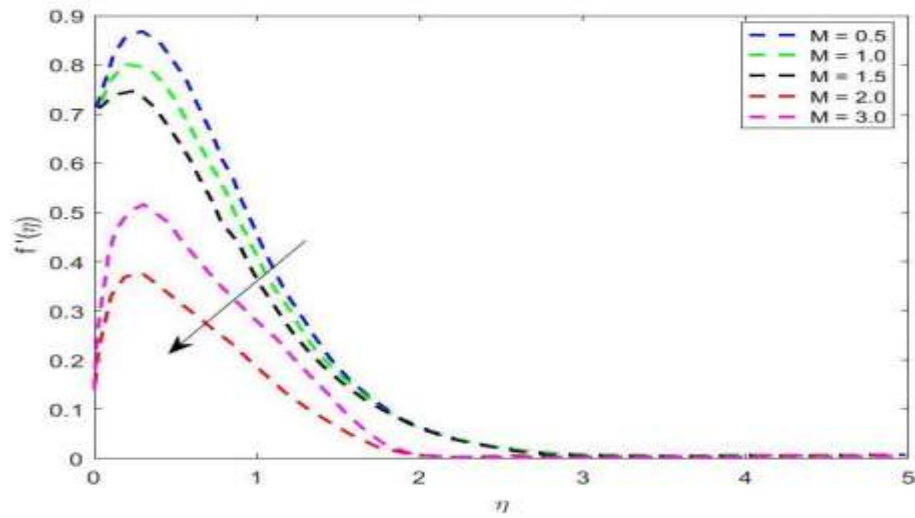


Figure 4.2: $f'(\eta)$ versus variable M

The fluctuation of the dimensionless velocity profile $f'(0)$ is shown in Figure 4.2. This transformation is brought about by varying values of the magnetic parameter M . There is some evidence that when M grows, the border layer area exhibits a general slowing of the velocity. The phenomenon may be understood by considering the Lorentz force, a resistive force generated by the applied MF and acting counter to the fluid's velocity. This indicates that the electrically conducting fluid encounters more flow resistance and dampening effects as the magnetic strength increases, as the momentum border barrier grows thinner.

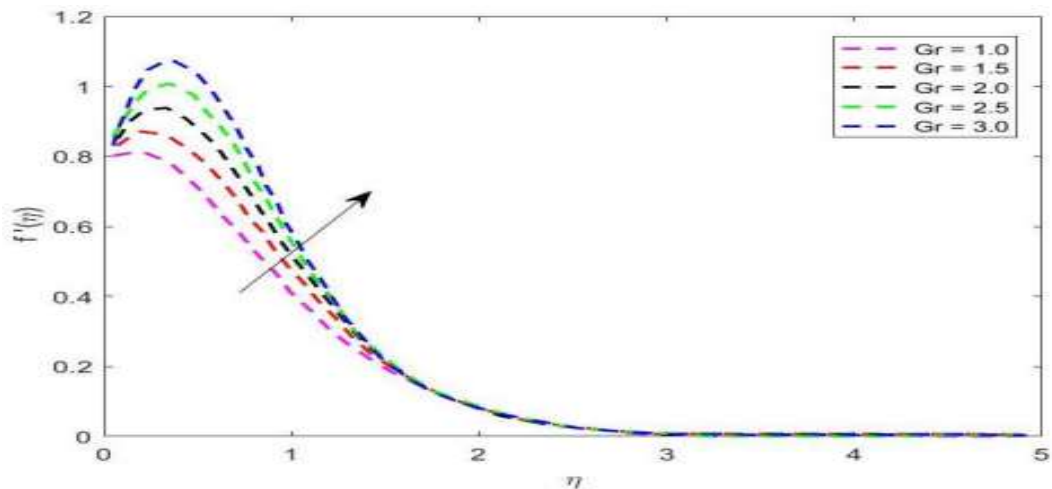


Figure 4.3: $f'(\eta)$ versus variable Gr

Figure 4.3 depicts the impact that the thermal Grashof number, denoted by the symbol G_{vt} , has on the patterns of velocity distribution. When vt is increased, the velocity profile is much improved, which is a considerable improvement. The higher the values of t , the stronger the thermal buoyant forces are in comparison to the viscous forces. This occurs as a result of the fact that t Expressions as a buoyancy force. The surface fluid flow is accelerated as a result of these buoyant forces, which in turn causes the velocity to increase & the momentum barrier layer to become denser. There is evidence to support the idea that heat convection is the primary process responsible for increasing fluid velocity.

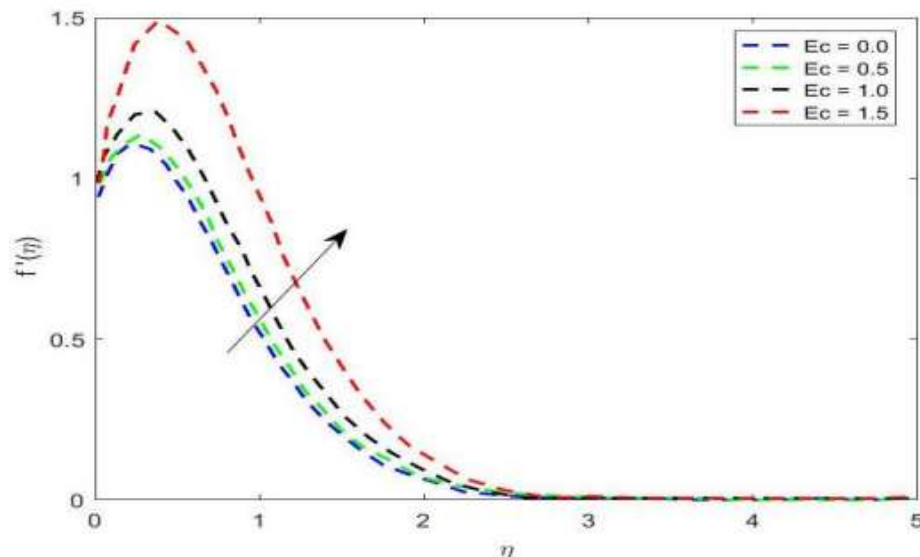


Figure 4.4: $f'(\eta)$ versus variable Ec

The page's Figure 4.4 illustrates the effect of the Eckert number c on the velocity profile. As the value of c rises, a little increase in velocity is seen. One measure of the amount of kinetic energy converted to internal energy via viscous energy dissipation is the Eckert number. The temp of the fluid increases in response to an increase in viscous dissipation, which in turn decreases the viscosity and quickens the flow rate. The velocity inside the border layer is somewhat increased as a result of this.

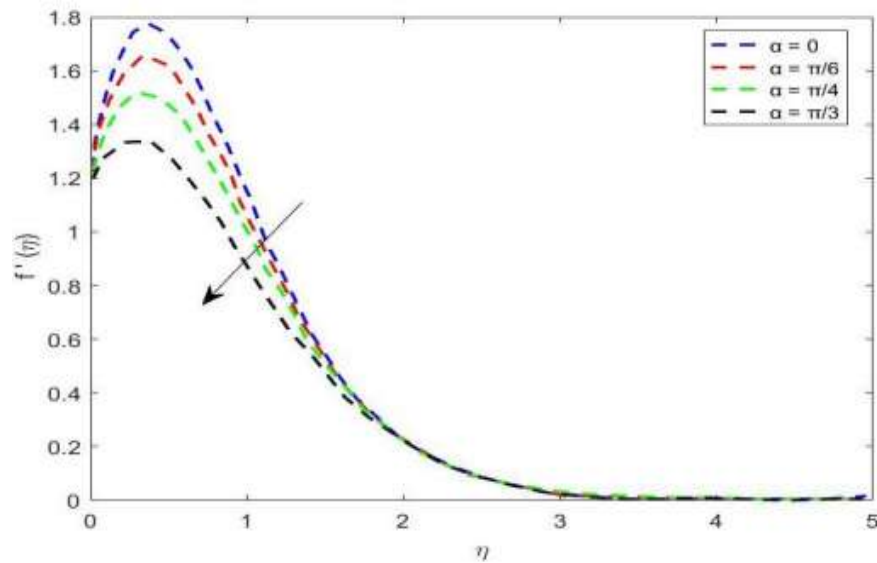


Figure 4.5: Velocity profile versus variable α

In Figure 4.5, it is shown that the velocity profile is shown to change depending on the values of the velocity slip parameter α . It should come as no surprise that elevating the value of α would result in an increase in the velocity in the vicinity of the surface location. Because the fluid has a lower coefficient of friction, it is feasible that it will glide more easily over smooth surfaces. This is because smooth surfaces are less likely to cause friction. Because the velocity gradient at the wall is smaller, the momentum at the border is also lower. This is because the wall is located at a lower elevation.

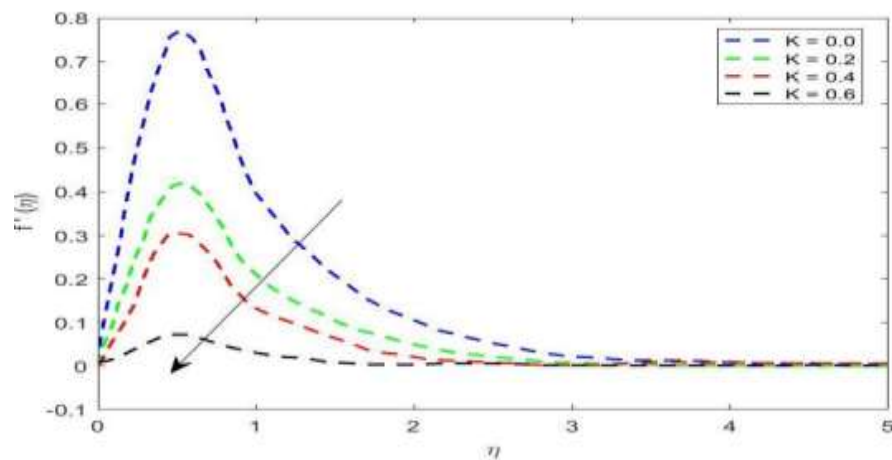


Figure 4.6: Velocity profile versus variable K

An illustration of the influence that the permeability parameter K has on the velocity distribution may be seen in Figure 4.6. The velocity across the border layer experiences a significant increase as a result of an increase in the value of K . When the permeability value is higher, it indicates that the fluid is able to pass through the porous medium with greater ease since it is met with less resistance. Increasing the thickness of the border layer is the result of lowering the drag forces that are present inside the medium and enhancing the velocity of the fluid.

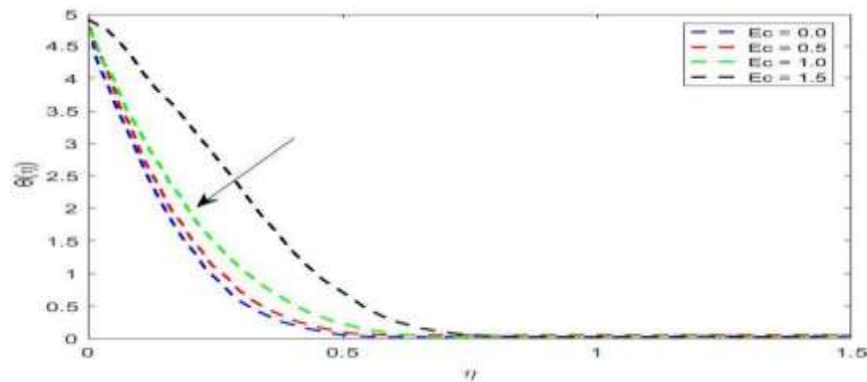


Figure 4.7: $\theta(\eta)$ versus variable Ec

An illustration of the influence that the Eckert number Ec has on the Temp Profile (η) may be seen in Figure 4.7. The temp is steadily rising, & this fact is becoming more obvious as time goes on. This result is brought about by the greater dissipation of viscous fluid, which resulted in the transformation of mechanical energy into thermal energy. It is because of this that the fluid creates a greater amount of heat, which results in a thermal border layer that is denser & a temp distribution that is more intense.

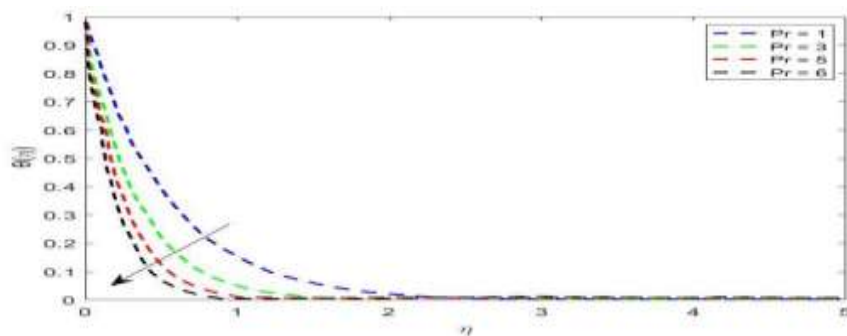


Figure 4.8: $\theta(\eta)$ versus variable Pr

Various Pr values to consider Figure 4.8 illustrates the Pr values as well as the temp profiles that are produced as a consequence, which differ depending on the Pr value. The temp distribution becomes more constrained as a consequence of an increase in Pr , which also leads to a commensurate reduction in the thickness of the thermal border layer. This is owing to the fact that fluids with higher Pr values have a lower thermal diffusivity, which means that heat cannot be dispersed from the surface of these fluids. This event is the result of this particular cause. The reason for the reduction in the overall temp field is due to this; heat is concentrated around the wall as a result of this.

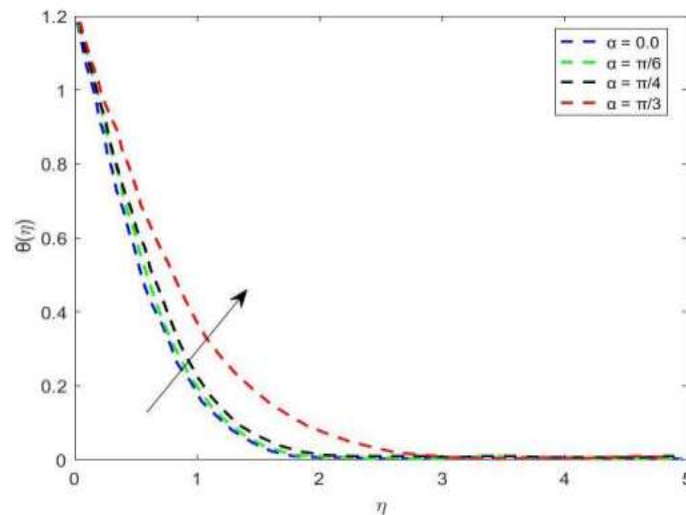


Figure 4.9: $\theta(\eta)$ versus variable A

Figure 4.9 provides both a visual illustration and an example of how the heat source/sink parameter A influences the temp profile. Both of these elements are included in the figure. When A is increased, the temp of the border layer will also increase as a consequence. When an internal heat source is present, the system receives an additional amount of thermal energy, which results in an increase in the temp distribution and an expansion of the thermal border layer to an even greater extent. This demonstrates without a shadow of a doubt that the flow's internal heat generation significantly enhances the flow's thermal qualities.

Table 4.1: Estimation of $-\theta'(0)$ and $f''(0)$, for the various parameters $Grt = 6$, $n = -0.65$, $\omega = 0.5$, $K = 1$, $Pr = 0.7$, $Ec = 0.2$.

Present Result					Mandal et al.			
M	Pr	δ_1	δ_2	α	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$
0.5	0.75	0.3	0.2	$\pi/6$	0.902920322	0.824503112	0.90292	0.82450
1	0.75	0.6	0.5	$\pi/6$	0.899965271	0.87451521	0.89996	0.87451
1.5	0.75	0.9	1.0	$\pi/6$	0.895731412	0.955973142	0.89573	0.95597

Figure 4.8 provides a visual depiction of the changes in temp profile that bring about variations in the Pr&tl number. These changes are brought about by variations in the Pr&tl number. In the event that there is an increase in the Pr values that have been observed, it would seem that the temp is lowering. It is expected that the value of Pr would rise, which will lead to a reduction in the thermal conductivity of the material. The temp profile has therefore decreased, as shown by what we have taken into consideration. There is a possibility that Figure 4.9 will illustrate the impact that the angle of inclination K has. By increasing the angle of inclination, the quantity of heat that can be transmitted from the fluid to the surface also rises. This is because the fluid is more inclined. It is possible to draw the conclusion, on the basis of the data shown in Table 4.1, that the Surface friction is a Expression that grows as the Thermal slip parameter increases. In addition to this, the Local Nusselt number displays a downward trend when the velocity slip is raised to higher levels.

4.5 CONCLUSION

Within the scope of this inquiry, a numerical analysis was carried out to explore the conduct of transfer of heats and fluid flow over an inclined plate that was subjected to free flow nanofluid flow via a porous composition. This investigation was conducted with the intention of determining the behaviour of the plate. All of the impacts of the MF, as well as the temp, viscosity, and inclination angle of the nanofluid, are taken into

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consideration in the survey. Every one of these individual facets is taken into consideration. In light of the results, a wealth of information on the influence of various physical parameters on the velocity and temp distributions of the border layer has been found.

The influence of the magnetic parameter on the motion field is one of the most important findings that can be brought up as a result of this research. When the magnetic parameter is increased, there is a discernible reduction in the velocity distribution that may be experienced. This behaviour might be explained by the Lorentz force, which functions as a resistive force and inhibits the flow of the electrically conducting nanofluid. This is one of the probable explanations for this conductivity. There exists an inverse relationship between the thickness of the momentum barrier layer and the intensity of the MF. Because of the relationship that exists between the two, this is the reason.

The findings of this research make it possible to draw a number of significant conclusions, one of the most significant of which is the influence that the magnetic parameter has on the velocity field. Increasing the magnetic parameter has been shown to result in a discernible reduction in the velocity distribution. This has been demonstrated statistically. This behaviour may have a number of plausible explanations, one of which is the Lorentz force, which functions as a resistive force and prevents the electrically conducting nanofluid from flowing freely. There is a possibility that the Lorentz force might provide an explanation for this behaviour. A possibility. When the intensity of the MF increases, the thickness of the momentum barrier layer will decrease. This is because the MF becomes stronger. This is because of the link that exists between the two, which is the reason there is this.

Among the many important discoveries that have been made as a result of this investigation is the influence that the magnetic parameter has on the velocity field, which represents one of the most important results. There is a discernible flattening of the velocity distribution that may be seen as the magnetic parameter is being raised. Another possible explanation for this phenomenon is the Lorentz force, which acts as a resistive force and inhibits the electrically conducting nanofluid from flowing freely.

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This force is a probable cause of this behaviour. This phenomenon might be explained by the Lorentz force, which is one of the probable explanations. When the strength of the MF is increased, the momentum barrier layer will get thinner as a consequence. These phenomena may be traced back to the connection that exists between the two causes.

Controlling the flow dynamics is mostly accomplished by the angle of inclination of the plate, which is the most important parameter. The buoyancy force component across the plate decreases as the angle of inclination rises, which leads to a decrease in the velocity of the fluid as it passes over the plate. The reason for this is because the plate has a more sharply inclined angle. The distribution of temp, on the other h&, exhibits the pattern that is anticipated, which is a temp profile that increases with increasing inclination angle. It seems that the temp profile is moving in an upward direction. The hypothesis that inclination has an effect on the buildup of thermal energy in the border layer is given more weight as a consequence of this discovery, which is a good outcome.

In addition to this, it has been shown that the Prandtl number has a considerable impact on the thermal border layer. In response to an increase in the Prandtl number, the temp profile becomes more uniform, and the thermal border layer becomes less dense. The Prandtl number demonstrates that this is a viable argument. Due to the fact that the thermal diffusivity of the nanofluid decreases as the Prandtl numbers increase, there is a limit to the amount of heat that may diffuse away from the surface. Due to this particular cause, this incidence is taking place.

There is a shift in the distribution of velocities and temp that takes place when there is a presence of a porous medium because it raises the barrier that the fluid that is flowing must navigate through. The quantity of fluid that is able to pass through a porous media is determined by its permeability, which directly influences the amount of heat that is transferred by convection. There is a decrease in fluid velocity and an increase in heat diffusion when there is an increase in the internal resistance inside the porous material.

In conclusion, the results of the simulations demonstrate that the flow & Heat exchange behaviour is significantly influenced by the intensity of the MF, the angle of inclination, the viscous dissipation, & the thermal characteristics of the nanofluid. These discoveries have important repercussions for the actual use of systems that make use of nanofluids & inclined porous surfaces. These systems include geothermal systems, heat exchangers, & thermal management applications.

MHD FLOW WITH SECOND-ORDER SLIP & RADIATION OVER A POROUS SHEET

5.1 INTRODUCTION

Researchers have shown a greater interest in the phenomenon of border layer fluid flow over surfaces that are increasing exponentially throughout the course of the last several years. In several technical applications for industrial reasons, the idea of an exponentially growing surface has been used in a number of different methods that have been utilised. The manufacture of fibres, nuclear reactors, & glass fibres are only some of the applications that fall under this category. These applications have made use of the idea of a surface that is increasing at an exponential rate. The most significant benefit that MHD flow provides is the capability to forecast the pace at which cooling occurs with it. The notion of magneto hydrodynamic flow, in combination with the energy equation of viscous fluid travelling over an exponentially increasing surface, is used in the manufacturing of gas turbines, as well as a variety of pieces of equipment for aeroplanes & a number of space crafts. At the time when Crane & colleagues. (1975) conducted their first investigation into the issue of linear stretching sheets, they concentrated their attention on the velocity of the border layer flow, which was determined by the distance that separated the two sites respectively. In order to find a solution to the problem of stagnation point flow across the stretched sheet, Ibrahim & colleagues. (2017) came up with an innovative proposal.

According to Irfan & colleagues.'s research from 2020, the problem of two-dimensional incompressible flow has to be resolved. According to the hypothesis put forward by Ishak & colleagues. (2011), the flow of nanofluid fluid is caused by a Chemical interaction that is influenced by the presence of a porous media. Majeed & colleagues. (2018) conducted an experiment with the purpose of determining how the processes of injection & suction influence the Casson fluid flow velocity. (N&eppanavar & colleagues., 2021) is the publication that contains the findings of the numerical solution of the nanofluid flow towards an exponentially growing surface. During the course of

their research, Ene & colleagues (2015) investigated the effects that heat production & absorption had on the two-dimensional viscous flow of nanofluids. It was hypothesised by Kumar & colleagues. (2017) that the exponentially expanding surface of the MHD nanofluid is the contributor to the unstable flow of the nanofluid. Under the circumstance of velocity slip & in the presence of porous medium effects, Bhattacharyya & colleagues (2011) conducted research on the Carreau nanofluid.

Aurangzaib & colleagues. (2021) conducted a study in which they investigated the effects of a surface-stretched nanofluid that was exposed to a Newtonian heating condition. Within the scope of their study, Bhattacharyya & colleagues. (2016) provided a comprehensive analysis of the impact that velocity slip circumstances have on Casson fluid flow. As stated in the research conducted by Mukhopadhyay & colleagues. (2013), Brownian motion & thermophoresis factors have the potential to influence the flow that occurs over a vertical sheet.

A research that was conducted by Waini & colleagues (2020) looked at the characteristics of a wide variety of parameters that have an impact on the flow across an exponential sheet. The system was designed by Nadeem & colleagues., 2020, with the purpose of addressing the problem of the viscous fluid model in an environment with porous media. Reddy & colleagues. (2021) conducted research on the flow of an incompressible fluid over a stretched surface while Chemical interactions were taking place. Mabood & colleagues. (2017) conducted research to investigate the impact that heat radiation has on the MHD of a fluid that is outside of the Newtonian regime. The Maxwell fluid flow problem, which included velocity slip, was resolved in the paper that was written by Nazeem & colleagues in 2020. In their research published in 2010, 2014, & 2013, Bhattacharyya & colleagues. investigated a number of issues that were associated with hydraulic flow by using analytical & numerical approaches. The heat source & sink effect was used in the research conducted by Mahabadi & colleagues. (2007) to investigate the radiative Heat exchange in nanofluid. Mabood & colleagues. (2016) conducted research with the purpose of determining the properties of nanofluids when they were exposed to a nonlinear stretching sheet. Researchers Ghosh & colleagues. (2018) were successful in finding a solution to the nanofluid issue by using the advanced finite element approach.

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A research that was carried out by Adegbie & colleagues (2015) investigated the impact that joule heating has on the Heat exchange that occurs in Maxwell fluid flows. An investigation of the ways in which homogeneity & heterogeneity influence the distribution of nanofluid flow velocities was carried out by Sheikholeslami & colleagues (2016) in their paper. A significant number of scientific articles have been devoted to the topic of nanofluid flow over an exponentially stretched sheet since the year 2016. In the study that Zeeshan & colleagues. (2018) conducted on axisymmetric casson fluid flow, Brownian motion & the thermophoresis effect were taken into consideration among other factors. In their 2017 study, Xun & colleagues proved the influence that viscous dissipation has on MHD flow. During their experiment, Raju & colleagues. (2016) followed the flow of fluids that were subject to free convection as they approached the point when they became stagnant. With slip conditions, Tlili & colleagues. (2019) create numerical & analytical solutions for unstable flows. These solutions are constructed for unstable flows. According to Shahzad & colleagues.'s 2020 study, the researchers investigated the heat source & sink impact in order to estimate the amount of heat transmission.

In the article "Mabood & colleagues., 2017," the authors address the influence that nanofluid flow has on porosity as well as its growth beyond the exponential sheet. In their 2017 publication, Pal & colleagues. presented a novel Nonlinear fluid modelmodel that has two different solutions. The influence of Soret & Dufour was investigated by Bhatti & colleagues. (2017) in the course of their research on Casson fluid flow. (Khalili & colleagues., 2017) made use of empirical data in order to illustrate the Darcy impact on the flow of thick fluids. The notion of ferrofluid flow in porous media was explained in depth by Sajid. Carreaunanofluid laminar flow across a vertical sheet was the subject of their investigation, & they have finished the appropriate analysis for it. The researchers S&hya & colleagues. (2018) looked at how the temp of the Casson fluid flow is impacted by the radiation conditions & chemical processes occurring in the environment. the existence of the two-dimensional incompressible flow that is induced by a stretched surface was proved by Ishak & colleagues. (2010).

Bhattacharyya & colleagues. (2013) investigated the convective Heat exchange in nano fluid by using the heat source & sink effect as the basis for their investigation. This was

done in order to better understand the processes involved. After the Lorentz force properties on flow field in the porous medium were discovered, Mukhopadhyay & colleagues. (2013) were able to recognise them. Following the disclosure of the characteristics, this action was taken

Within the scope of this chapter, the concepts of MHFT and RTH with a second-order slip border condition are further explored. An exponentially expanding sheet is used in order to depict the flow of the fluid. It is important to take into consideration both the heat conductivity and the impact of the MF. The similarity transformation is used to build an ODE system from a theoretically determined nonlinear flow description after the system has been formed. There is a numerical solution found for the higher-order nonlinear ordinary DE that are associated with this. When taking into mind first- and second-order slip parameters, the velocity and temp profile are taken into consideration. The skin friction and the Nusselt number coefficient both exhibit significant functional changes in relation to the magnetic parameter. Some of these fluctuations are rather considerable. When it comes to a wide range of important concerns, graphs are used to investigate the several aspects of the flow. A decrease in the Local Nusselt number and an increase in the magnetic parameter both contribute to an increase in the skin friction coefficient, as shown by the data. As the Prandtl number increases, the temp field decreases, but the radiation parameter causes the transfer of heat to increase. This is because the radiation parameter is larger. The primary purpose of this investigation is to investigate the ways in which the existence of second-order slip affects the flow of MHD fluid across a sheet that has been stretched exponentially. In this study, experiments are conducted to investigate the impacts of radiation parameter, magnetic parameter, first-order slip, & second-order slip. The findings of this work have significantly contributed to our existing understanding of border layer flow issues in porous media. Numerical solutions to the following mathematical problems may be obtained via the use of the bvp4c module in Matlab. In the course of our work, we have developed a graphical representation of the fluid velocity & temp profile for a number of important factors. The findings of this research provide weight to the findings of past investigations & are consistent with those findings.

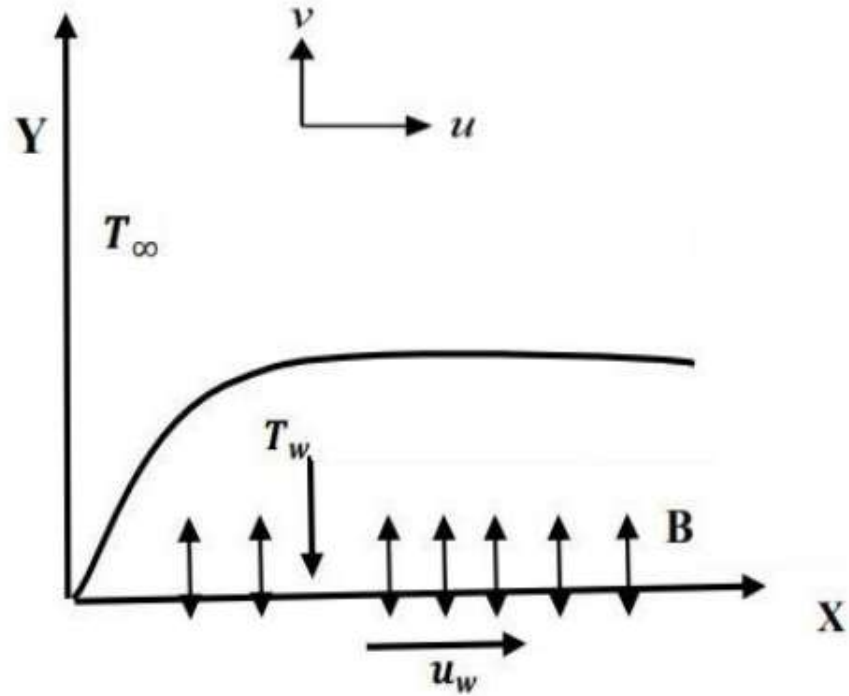
Examining the flow of MHD fluid over an exponentially stretched sheet in the presence of second-order slip is the objective of this research project. During the course of this study, we have looked at the effects of first- & second-order slip, as well as the radiation parameter & the magnetic parameter. A number of important discoveries about border layer flow difficulties in porous media are presented in this new study. With the assistance of bvp4c, the numerical approach in Matlab is used in order to solve the accompanying mathematical problems. This analysis provided a graphical representation of the temp& velocity curve of the fluid for a very large number of important factors. According to the results of the main study, it is in conformity with & supports the hypothesis.

5.2 MATHEMATICAL FORMULATION

As time goes on, the 2-dimensional magneto hydrodynamic fluid flow approaches that of an exponentially stretched sheet. When taking the stretching sheet, the x coordinate sometimes called the x axis is used. The value of x equals zero on the x -axis. A known. MF is assumed to have a strength of $B = B_0 e^{x/2L}$, with B_0 being considered constant. It is believed that the direction of the MF is perpendicular to the axis of rotation. One may argue that the induced MF doesn't matter much when a Reynolds number is small.

Assuming that the thermal conductivity of a material varies in a linear fashion with temp, this situation is presented. $u_w = -C_e x L$ refers to velocity at border, where C is constant, $v = v_w = v_0 e^{x/2L}$ refers to special kind of velocity, where v_0 is a constant, v_0 is considered as velocity suction for $v(x) > 0$, when $v(x) < 0$, v_0 is the velocity blowing, $T = T_w = T_{\infty} + T_0 e^{x/2L} + N \Theta T$ refers to fluid temp. N is the thermal. slip coefficient, F_w is the temp of the fluid at the wall, T_{∞} is the temp of the ambient environment, F_0 is the temp of the reference, and F_0 is the temp of the fluid. Regarding the flow issues involving the border layer, the continuity, momentum, and

energy equations are controlled in the following manner: Charles M. I. Char (1988) and



Chiam (1998)"

Figure 5.1: Modelling the flow physically

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu u}{k} - \frac{\sigma B^2}{\rho} u \quad (5.2)$$

$$\rho c_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k_0 \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (5.3)$$

The variables a & b are both included as components of the fluid velocity along the x -axis, where the former is considered as an independent variable & the latter as an additive one. The following symbols are used in mathematical notation: ν for kinematic viscosity, k for coefficient of permeability, σ for Chemical interaction rate constant, ck for specific heat, T for fluid temp, k_f for thermal conductivity, ρ for fluid density, & ca for radiative heat flux. $B = B_0 e^x / 2L$ is the formula for the MF, with B_0 being the constant representing the MF.

For this fluid model to work, the following restrictions must be satisfied:

$$u_w = -C e^{x/L} \quad (5.4)$$

$$\begin{aligned} u_{slip} &= \frac{2}{3} \left(\frac{3-\alpha l^3}{\alpha} - \frac{2}{3} \frac{1-l^2}{k_n} \right) \lambda \frac{\partial u}{\partial y} - \frac{1}{4} \left(l^4 + \frac{2}{k_n^2} (1-l^2) \right) \lambda^2 \frac{\partial^2 u}{\partial y^2} \\ &= a_1 \frac{\partial u}{\partial y} + a_2 \frac{\partial^2 u}{\partial y^2} \end{aligned} \quad (5.5)$$

$$u = u_w + u_{slip} = -C e^{x/L} + a_1 \frac{\partial u}{\partial y} + a_2 \frac{\partial^2 u}{\partial y^2} \quad (5.6)$$

$$v = v_w = v_0 e^{x/2L}, T = T_w = T_\infty + T_0 e^{x/2L} + N \frac{\partial T}{\partial y} \quad \text{at } y = 0 \quad (5.7)$$

$$u \rightarrow 0, T \rightarrow T_\infty \quad \text{at } y \rightarrow \infty \quad (5.8)$$

The symbol kn represents the Knudsen number, α represents the momentum adaption coefficient, & λ represents the molecular mean free path parameter. l is equal to the minimum of $[1 / kn, 1]$, where L is the typical length. Both $X1$ & $X2$ remain unchanged.

5.3 METHOD

Taking similarity transformation by using Expression $\psi(x, y)$ to simplify the border value problem

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (5.9)$$

The stream Expression uniformly satisfies Eq. (5.1), & Eq. (5.2) reduced to

$$\frac{\partial \psi}{\partial y} \cdot \frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} - v \frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B^2}{\rho} \frac{\partial \psi}{\partial y} \quad (5.10)$$

Where $B = B_0 e^{x/2L}$

Using Rossel& approximation

$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y}$, T^4 is linear temperature function, where k^* is mean absorption coefficient, σ is Stefan Boltzman constant, the function T^4 expanded in Taylor's series and ignoring the higher terms. We have

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

Therefore Eq. (5.3) becomes

$$\rho c_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k_0 \frac{\partial^2 T}{\partial y^2} - \frac{16\sigma T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad (5.11)$$

Using stream Expression Eq. (5.11) becomes

$$\rho c_p \left[\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right] = k_0 \frac{\partial^2 T}{\partial y^2} - \frac{16\sigma T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad (5.12)$$

Where

$$\psi = \sqrt{2\nu L C} f(\eta) e^{x/2L}, \quad \eta = y \sqrt{\frac{C}{2\nu L}} e^{x/2L}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad (5.13)$$

For the similarity solution permeability is assumed to have the following form

$$k(x) = 2k_0 e^{-x/L} \quad (5.14)$$

Where k_0 is reference permeability

A fluid's thermal conductivity changes in a straight line as the temp rises.

$$k_f = k_\infty (1 + \epsilon \theta) \quad (5.15)$$

The parameter ϵ represents the variance in thermal conductivity.

Equations (5.10) & (5.12) are altered by using the similarity transformation & border layer approximation.

$$f'''' + f f'' - 2f'^2 - \frac{f f'}{\kappa} - M f' = 0 \quad (5.16)$$

$$\left(1 + \frac{\epsilon}{3} R\right) \theta'' + \text{Pr}(f \theta' - \Gamma \theta) = 0 \quad (5.17)$$

With border conditions

$$f'(0) = 1 + \gamma f''(0) + \delta f'''(0), f(0) = S, \theta(0) = 1 + \alpha^* \theta'(0) \quad \text{as } \eta = 0 \quad (5.18)$$

$$f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (5.19)$$

Where $M = \frac{2\sigma B_0^2 L}{\rho C}$ is magnetic parameter, $R = \frac{4\sigma T_\infty^3 L}{k k^*}$ is radiation parameter, $Pr = \frac{\rho C_p}{k_\infty}$ is prandtl number,

$\gamma = a_1 \sqrt{\frac{C}{2\nu L}} e^{x/2L}$ is first order slip parameter and $\delta = a_2 \left(\frac{C}{2\nu L} e^{x/L} \right)$ is second order slip parameter. $\alpha^* = N \sqrt{\frac{C}{2\nu L}} e^{x/2L}$ considered as thermal slip parameter for the flow problem.

These are the physical characteristics that are of importance, such as the Wall shear coefficient & the Nusselt number for fluid flow, which we get by applying the similarity transformation described above.

$$C_f = \frac{\tau_w}{\rho u_w^2} \quad \text{where the wall shear stress } \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (5.20)$$

$$Nu_x = \frac{x q_w}{k(T_w - T_\infty)} \quad \text{where the wall heat flux } q_w = -R \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (5.21)$$

$$C_f = \sqrt{\frac{x}{2L}} Re_x^{-1/2} f''(0), \quad Nu_x = -\sqrt{\frac{x}{2L}} Re_x^{-1/2} \theta'(0) \quad (5.22)$$

Where $Re_x = u_w x / \nu$ represents the Reynolds number

The mathematical simulation of the flow issue, which is represented in the nonlinear system of ODE (5.16) to (5.17), together with the border conditions that correspond to it (5.18) and (5.19). It is believed that the MATLAB software bvp4c is suitable for solving issues of this kind since it allows for the estimated result to be obtained. Through the use of the finite difference technique, the software is able to resolve the border value problem. Through the application of the following transformation to the changed equations, we are able to convert the issue into an initial value problem.

$$f' = z, \quad z' = p, \quad p' = Mz + \frac{z}{K} + 2z^2 - fp \quad (5.23)$$

$$\theta' = q \quad q' = -\frac{pr(fq - z\theta)}{(1 + \frac{4}{3}R)}$$

& the transformed dimensionless border conditions become

$$f(0) = S, (0) = 1 + \gamma p(0) + \delta p'(0), \theta(0) = 1 + \alpha q(0). \quad (5.24)$$

$$z(\infty) = 0, \theta(\infty) = 0.$$

The border condition is regarded as a finite value, denoted as $\eta \rightarrow \infty$, with $\eta_{\max}$ being given a value of 40. Taking into consideration the numerical scheme, an appropriate interval is assessed to be $\nabla\eta = 0.01$, with the range of tolerance being considered up to 10^{-5} . It was necessary to modify the trial values of $f''(0)$ & $\theta'(0)$ in order to get a more accurate approximation while still satisfying the accompanying border condition.

5.4 RESULTS

In the present numerical investigation, the border condition at infinity is approximated by assigning a finite computational domain, represented by $\eta \rightarrow \infty$. For numerical implementation, the maximum value of the similarity variable is fixed at $\eta_{\max} = 40$. This is shown to be enough to guarantee that the temp& velocity profiles converge to their asymptotic values. If you go with this option, you can be certain that the numerical domain will properly accommodate the imposed border constraints at infinity.

The numerical scheme is executed using a step size of $\nabla\eta = 0.01$, which provides a suitable balance In relation to computational efficiency & solution accuracy. Furthermore, the convergence criterion is set by imposing a tolerance limit of 10^{-5} for all dependent variables. These numerical parameters are carefully selected to ensure the stability & reliability of the obtained solutions throughout the computational process.

To achieve a more accurate approximation of the governing border value problem while strictly satisfying the associated border conditions, appropriate adjustments are made to the initial trial values of $f''(0)$ & $\theta'(0)$. These shooting parameters are iteratively refined until the numerical solutions converge within the prescribed tolerance level. The results obtained through this procedure form the basis for

analyzing the effects of various physical parameters on the flow & Heat exchange characteristics discussed in the subsequent sections.

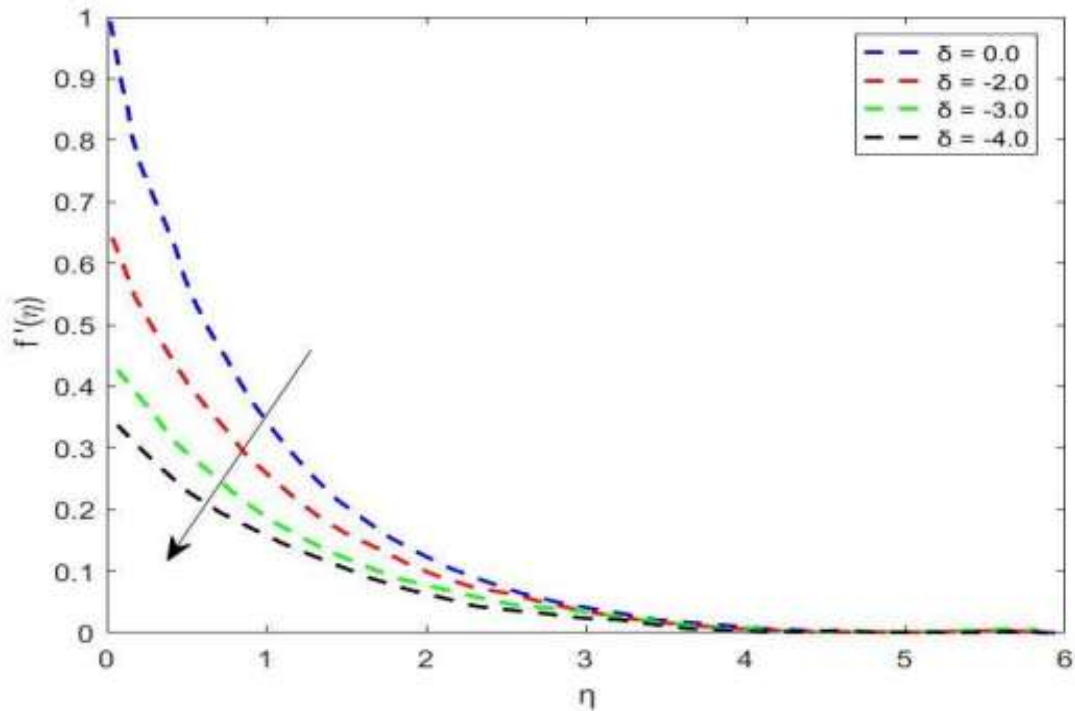


Figure 5.2: $f'(\eta)$ versus δ for $\gamma = 0$, $M = 0.2$, $R = 0.7$, $Pr = 0.5$, $\alpha = -2$

The visual representation in Figure 5.2 illustrates the influence of the slip parameter δ on the dimensionless velocity profile $f'(\eta)$ for the values of $\gamma = 0$, $M = 0.2$, $R = 0.7$, $U_f = 0.5$, and $\alpha = -2$. Several recent investigations have shown that fluctuations in the value of δ have a significant influence on the distribution of velocities within the border layer structure. There is a correlation between increasing the value of the slip parameter and the velocity near the wall, which indicates that there is less surface friction. Rather than entirely adhering to the no-slip criterion, which would ordinarily prevent this from happening, the fluid is allowed to partially glide over the surface owing to the slip condition. This is the reason why this is the case. The consequence of this is that surface-localized velocity gradients are minimised, and the momentum barrier layer becomes smaller. It is essential to take slip effects into consideration when regulating flow resistance, and this behaviour becomes much more noticeable in circumstances that include flow configurations on the micro- and nano-scale dimensions.

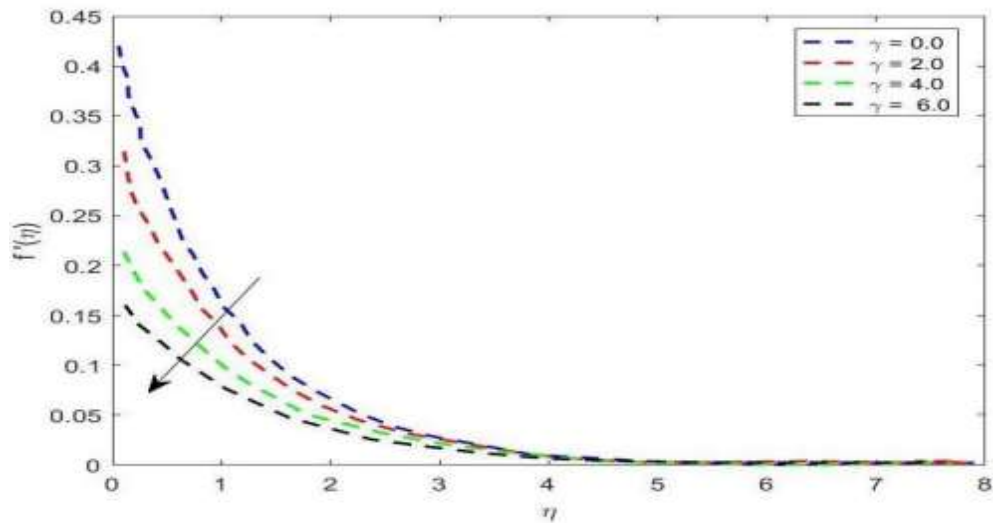


Figure 5.3: $f'(\eta)$ versus γ for $\delta = -1$, $M = 0.2$, $R = 0.7$, $Pr = 0.5$, $\alpha = -2$

Figure 5.3 illustrates how the velocity profile $f'(\eta)$ changes over time. The variables $\nu = -1$, $M = 0.2$, $R = 0.7$, $PZ = 0.5$, & $\beta = -2$ determine this fluctuation via the stretching/shrinking parameter γ . The graphic makes it quite evident that as γ grows, the velocity profile becomes much better. Increasing the stretching rate increases the surface drag force on the fluid, which in turn propels the fluid particles in the direction of flow, according to physical theory. What this means is that the momentum border layer becomes thicker & faster as it moves along the border layer. It is suggested that the stretching parameter is very important for changing the fluid's speed & improving momentum transfer, & our result supports that theory.

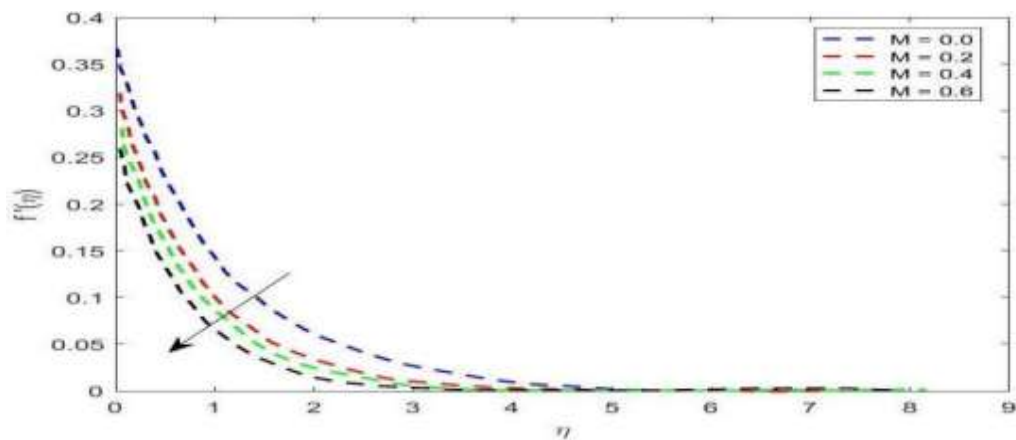


Figure 5.4: $f'(\eta)$ versus M for values $\gamma = 0, 8$, $\delta = -1$, $R = 0.7$, $Pr = 0.5$, $\alpha = -2$

The impact that the magnetic parameter M has on the velocity distribution $f'(\eta)$ is shown in Figure 5.4, which displays the values that are being taken into account. In the event that the magnetic parameter is increased, it has been shown that there is a decrease in the velocity profile that is seen over the border layer. It has been noted that this is something that has occurred lately. There has been a conclusive observation made in this regard. The application of a MF causes the formation of a Lorentz force, which acts in the opposite direction to the direction in which the fluid is moving. This force is a consequence of the application of the MF. The result of this force is the behaviour that you see here. Because of the resistive character of this force, the flow velocity is slowed down, and the thickness of the momentum barrier layer is significantly reduced. Both of these effects are quite significant. Neither of these impacts is negative in any way. The MF has the potential to be used as an efficient regulation mechanism for the purpose of lowering the velocity of the fluid in flows that contain electrical conductivity. This is a result of the fact that the MF has the capacity to be employed.

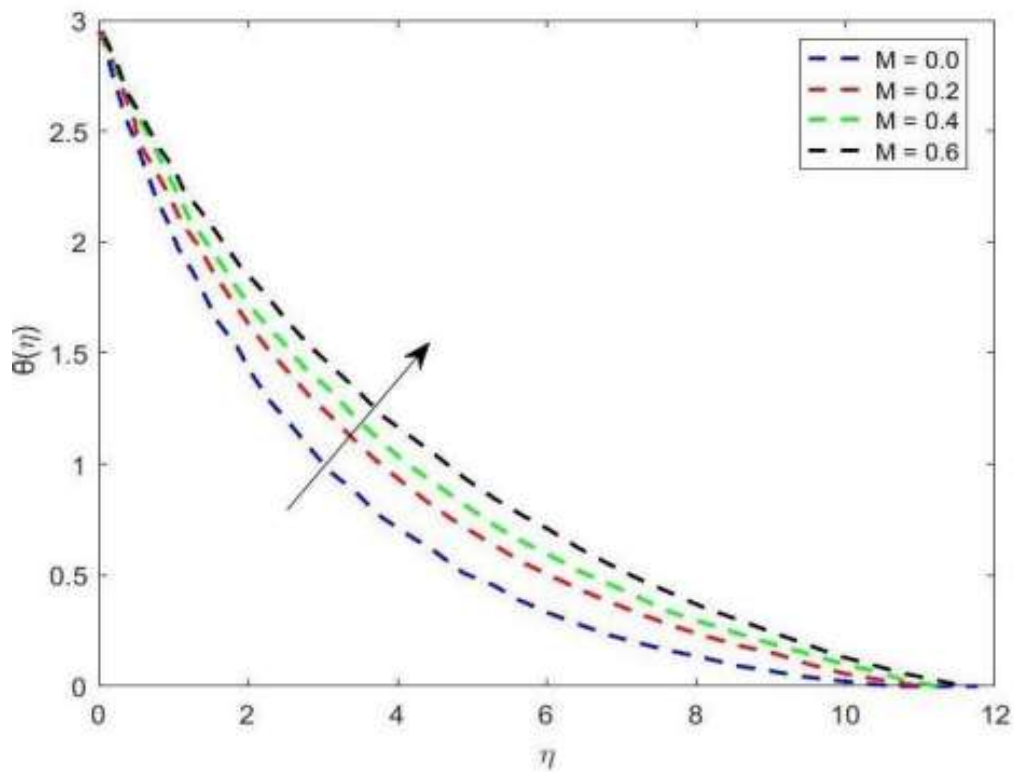


Figure 5.5: (η) versus M for $y = 0,8 = -1$, $R = 0.7$, $Pr = 0.5$, $\alpha = -2$

Keeping the values and $a = -2$ constant, Figure 5.5 illustrates the fluctuation in the temp profile (θ) for various values of the magnetic parameter A . This is done while maintaining the same values. It has been shown that the temp distribution inside the border layer grows in a manner that is proportional to the increase in the magnetic parameter. The generation of a MF causes an increase in the resistive force, which in turn sets off the Joule heating process and converts kinetic energy into thermal energy.

This leads to an increase in the temp of the surface as well as an increase in the thickness of the thermal border layer. Based on this, it seems that the MF has a dual effect: it slows down the fluid while simultaneously increasing the heat energy that is contained inside it.

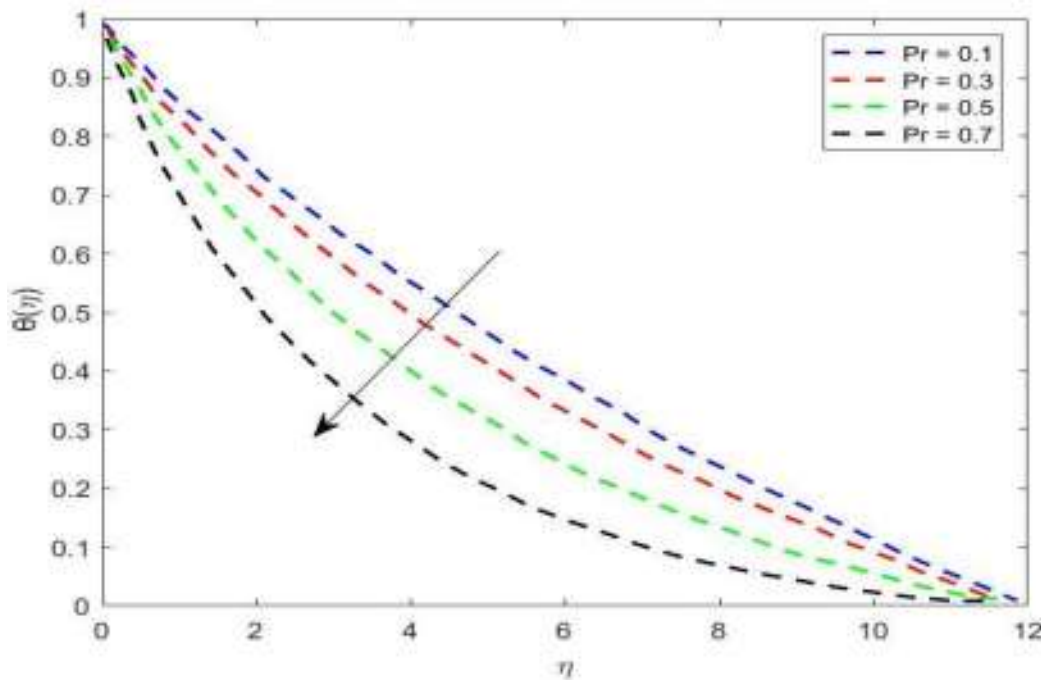


Figure 5.6: (η) versus Pr for $y = 0$, $\delta = -1$, $M = 0.2$, $R = 0.7$, $\alpha = -2$

As shown in Figure 5.6, the influence of the Prandtl number Pr on the temp profile $\theta(\eta)$ is shown for the values of y for 0 degrees, & for -1 degrees, M for 0.2 degrees, R for 0.7 degrees, and a for -2 degrees. A decrease in the thickness of the thermal border layer and a decrease in the temp distribution are both shown to be the results of an increase in the value of Pr , as seen in the figure. A higher Prandtl number is comparable to a lower thermal diffusivity in the physical world. This implies that it

restricts the transport of heat away from the surface. In other words, it is a measure of how readily heat can be transferred. Because of this, heat is limited closer to the wall, which leads to temp gradients that are steeper and a decrease in the thickness of the thermal border layer.

As a consequence of this, the temp gradients are steeper. Due to the fact that the Prandtl values of fluids such as oils are often higher, this behavior is considered to be typical for fluids such as oils.

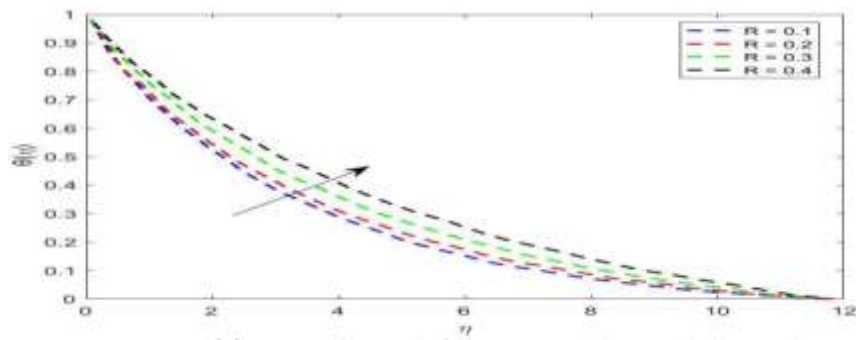


Figure 5.7: (η) versus R for $\gamma = 0, \delta = -1, M = 0.2, Pr = 0.5, \alpha = -2$.

In Figure 5.7, we can see how the radiation parameter R affects the temp profile (η) , where $\gamma = 0, \nu = -1, M = 0.2, Pf = 0.5, \&\alpha = -2$ continuously. Raising the value of X leads to a much more uniform distribution of temp within the border layer. This effect is brought about by thermal radiation, which raises the temp of the fluid by transferring more energy to it. The overall temp rises because the radiative heat flow increases, making the thermal border layer denser. This finding highlights the importance of thermal radiation in applications involving high-temp flows.

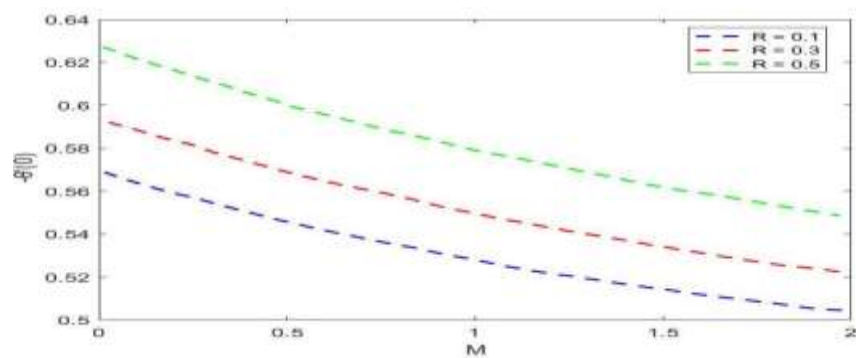


Figure 5.8: $\theta' (0)$ versus R for $\gamma = 0, \delta = -1, M = 0.2, Pr = 0.5, \alpha = -2$

A representation of the fluctuation of the wall temp gradient $\theta'(0)$ in relation to the radiation parameter g is shown in Figure 5.8. When the value of y is equal to zero, β is equal to minus one, M is equal to 0.2, Pr is equal to half, and β is equal to minus two, this variation occurs. As the value of R increases, it is evident that the magnitude of $\theta'(0)$ decreases in parallel with this change. With this information, we may deduce that the rate of heat transmission over the surface is decreasing and has slowed down. In the actual world, there is a smaller temp differential between the surface and the fluid. This is because an increase in thermal radiation leads the fluid that is close to the wall to become hotter. There is a decrease in the rate of heat conduction from the surface to the fluid as a consequence of this effect. When it comes to the building of thermal systems, this insight is very essential, especially in situations where radiation impacts are significant.

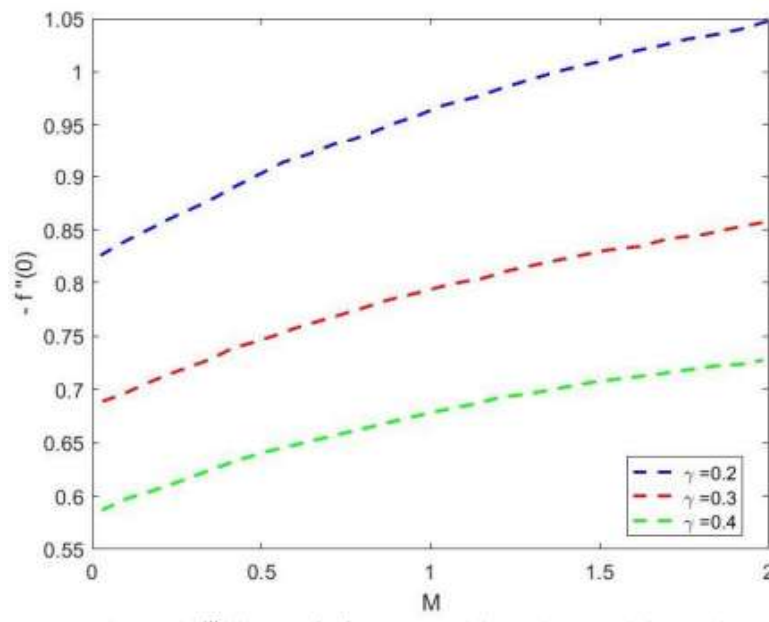


Figure 5.9: $f''(0)$ versus for $\delta = -1$, $M = 0.2, 0.7, Pr = 0.5$, $\alpha = -2$.

The Expression of the Nusselt number (η) for the modified magnetic parameter (M) in conjunction with the Radiation parameter (R) is established in Figure 5.8. As the graph demonstrates, the Nusselt number is exhibiting a decreasing tendency, which is followed by a rise in the values of M & W . This pattern is shown by the graph. On display in Figure 5.9 is a contrast In relation to the first-order slip effect & the manner in which the Surface friction $d(0)$ is shown. The fact that the graph of $X(0)$ goes

through a considerable modification as a consequence of X & X is something that must be acknowledged completely. The graph offered a concise & understandable explanation of how the value of $f(0)$ increased as the value of f increased. It was necessary to do a comparison In relation to the data & the literature that had been published in the past in order to ensure that the approach & the findings were accurate.

As an additional feature, Table 5.1 includes a numerical estimate of the outcomes of $-\theta'(0)$ when it is exposed to the impact of Pr values. The findings that were obtained & the results that were published by Sandhya & colleagues., 2021 are very commensurate with one another. There is a high degree of concordance In relation to the two.

Table 5.1: Numerical estimation of $-\theta'(0)$ for the selected Pr&tl number Pr when $\gamma = 0, \delta = -1, M = 0.2, R = 0.7, \alpha = -2$.

Pr	Sandhya et. al.	Ishak et. al.	Present Result
1	0.954784	0.9548	0.95478392
3	1.869073	1.8691	1.86907423
4	2.204507	2.5001	2.20450691

5.5 CONCLUSION

Understanding the border layer flow of an electrically conducting fluid over a sheet that is spreading at an exponential rate is the key goal that we have set for ourselves throughout this investigation. There has been a significant amount of study conducted on the implications of second-order velocity slip on flow and transfer of heat dynamics. The thermal radiation, the Prandtl number, the slip parameters of the first and second order, and the MF are some of the things that are taken into consideration in this extensive study including a significant lot of information. The generation of numerical solutions and the subsequent display of those solutions may result in the generation of a graphical representation of the behaviour of the velocity and temp profiles under a variety of different physical situations.

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An examination of the velocity distribution as a Expression of the second-order slip parameter is one of the significant discoveries that have been made as a result of this working. The retrograde tendency of the velocity profile $f'(\eta)$ is more visible when the second-order slip parameter is set to $\delta = -4$. This is in contrast to the situation when $\delta = 0$ is used. This discovery is the result of painstaking monitoring of the situation. The slowness might be explained by a rise in surface resistance, which would cause the fluid's mobility to decrease & the strength of the energy border layer to increase. Both of these factors would explain the slowness by itself.

In addition, an investigation has been carried out in order to ascertain the impact that the first-order slip parameter has on the velocity profile that is being investigated. Whenever the value of γ is equal to zero, which is the condition that corresponds to the traditional no-slip criteria, the velocity $f'(\eta)$ reaches its highest possible value. The maximum potential value that may be accomplished is represented by this very outcome. On the other h&, when the slip parameter is increased to a value of $\gamma = 6$, there is a noticeable decrease in the velocity profile that can be seen. It is true that this is the situation. The fact that this is the case makes it abundantly evident that a greater slip at the surface reduces the amount of contact that the fluid has with the stretched sheet, which ultimately results in a significant decrease in flow velocity. It is because of the connection that exists In relation to the two that this is the case.

Since the velocity profile $f'(\eta)$ is much higher when M is set to zero & significantly lowers when M is raised to 0.6, it is obvious that the magnetic parameter has a significant influence on the flow field. This is because the velocity profile is significantly greater when. It is always the case that this happens when the magnetic parameter is 0. It is possible that the resistive Lorentz force that is induced by the applied MF is the source of this slowing. Due to the fact that this force is Expressioning as a counterweight to the velocity of the fluid, momentum cannot be transmitted inside the border layer.

When compared to the behaviour of the velocity, the temp distribution (η) has a tendency to increase in proportion to the change in the magnetic parameter. The velocity & this occur at different rates than one another. The temp profile continues to

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be lower than it was previously when the value of M is equal to zero. On the other h&, when the value of M is 0.6, the temp distribution seems to be far more expansive. It is possible that this rise in temp might be explained by the fact that the total quantity of thermal energy that is stored within the fluid increases as a consequence of the resistance that is induced by the MF.

Concerning the influence that the Prandtl number has on the temp profile, careful thought and investigation have been carried out. According to the findings of the research, the temp distribution (n) achieves its greatest value when the Prandtl number is less ($Pr = 0.1$), and therefore, it decreases as the Prandtl number increases to 0.7. When Prandtl numbers are increased, there is a drop in thermal diffusivity, which eventually results in a thermal border layer that is thinner. This impact is brought about as a result of this decrease.

As an additional point of interest, in contrast to the Prandtl number, the radiation parameter exhibits a trend that is in the opposite direction. There is a link between a rise in the radiation parameter and an augmentation in the temp profile (n), which implies that thermal radiation contributes more heat energy to the flow field. This is because the parameters are related to one another. The significance of this influence is amplified in circumstances when the temp is high and radiative transfer of heat is the major route of heat transmission.

The numerical results show that the flow &Heat exchange properties over an exponentially stretched sheet are strongly affected by the second-order velocity slip, the strength of the MF, the thermal radiation, & the fluid properties. Polymer extrusion, electronic component cooling, & MHD-based thermal structures are just a few examples of the many industrial & technical domains where this study's results might be useful. The significance of slip effects &MFs cannot be overstated in any of these cases.

CONCLUSION

In this thesis, the major purpose has been to offer a complete mathematical & numerical research of the transport of basic physical characteristics, namely momentum, heat, & mass, in Newtonian fluid flow under the impact of many interacting physical processes. This inquiry has been carried out in order to achieve the primary objective. This work gives a better knowledge of how complex border layer phenomena develop when many governing factors operate concurrently. This is accomplished by merging sophisticated fluid mechanics theory with rigorous numerical approaches. The formulation of the issue takes into consideration actual physical assumptions, which enables the analysis to be both theoretically rigorous & practically useful for applications in contemporary engineering.

Fluid flow across a range of stretched & inclined geometries, which are commonly encountered in industrial processes & technical systems, has been the subject of the inquiry that has been carried out. Applications such as polymer extrusion, surface coating, heat processing, & aerodynamic design are examples of areas in which certain geometrical configurations are especially significant. The research shows the importance that surface geometry plays in determining momentum transfer & thermal properties inside the border layer. This is accomplished by taking into account both stretching & inclination effects, which allows for a more comprehensive understanding of the physical behavior studied.

In the course of this inquiry, one of the key focuses has been on magnetohydrodynamic (MHD) flow. This kind of flow is distinguished by the significant change in flow behaviour that takes place as a consequence of the interaction In relation tomagnetofluids (MF) & electrically conducting fluids that are introduced from the outside. Forces known as Lorentz forces are generated when a MF is present. These forces are characterised by their ability to impede the movement of fluids. Controlling the flow of fluids may be accomplished via the use of this approach. MFs are used in a variety of technical areas, including electromagnetic pumps, MHD generators, nuclear reactor cooling, & liquid metal processing, all of which include MFs to control the

velocity of fluids & the Heat exchange. This particular aspect of the study is very relevant to these technological domains.

The research also takes into account the circumstances of velocity slip at the interface between the fluid and the solid, going beyond the traditional assumption of no slip, which often fails in applications involving micro- and nano-scale flow. The inclusion of slip effects enables a more realistic depiction of flow behavior in contemporary microfluidic and nanofluidic systems. Slip effects become more relevant as characteristic length scales shrink, and their inclusion allows for this representation. Based on the findings of the research, it is clear that velocity slip plays a significant part in lowering surface shear stress and changing velocity distributions along the wall.

A thorough incorporation of the impacts of porous media into the mathematical model has been carried out in order to take into account the flow resistance and energy transfer that takes place inside structures that are permeable. Porous materials are used widely in a wide range of applications, such as geothermal engineering, filtration systems, thermal insulation, and energy storage devices, amongst others. According to the results, which reveal that permeability has a considerable influence on both the velocity of fluids and the temp profiles of fluids, porous media have the potential to be good instruments for managing fluid flow and enhancing thermal management performance. This is shown by the fact that porous media have the potential to be excellent instruments.

When radiative Heat exchange becomes significant, the energy equation has been updated to incorporate thermal radiation in order to handle flow situations that include high temp. This modification was made for the aim of addressing various flow scenarios. There are some applications in which radiation effects are of enormous consequence. Some examples of these applications are solar thermal collectors, high temp reactors, & industrial furnaces. Not only does thermal radiation increase heat transmission inside the border layer, but it also has a significant impact on temp distributions, as shown by the results of the numerical analysis. This is especially true when combined with the effects of magnetic & porous materials.

The phenomenon known as mixed convection, which takes place when both forced & free convection are present at the same time, has been subjected to extensive research

in order to achieve the goal of accurately reflecting the combined influence of buoyant forces & external flow. It is common for this phenomena to take place in a broad range of practical systems, including the cooling of electronic equipment, air fluxes, & heat exchangers that Expression at low velocities. The results of the study make it abundantly clear that buoyant forces have a considerable interaction with flow fields that are applied, which leads to complicated changes in velocity & temp profiles. This is obvious from the findings of the research.

To make the governing equations simpler, we transformed the nonlinear PDE into a system of connected ordinary DE using appropriate similarity transformations. To make the equations easier to underst&, this was done. This transformation technique facilitates the discovery of an efficient numerical solution while also assisting in the preservation of the issue's basic physics. Success in using similarity analysis as a tool in border layer theory demonstrates the tool's importance.

The system of ODE that was produced as a consequence was solved numerically by using trustworthy & well-established computing methods. It has been proved beyond a reasonable doubt that numerical approaches are useful in dealing with highly nonlinear border layer issues. This is especially true in circumstances when analytical solutions are either exceedingly difficult to get or completely unreachable. Within the scope of this investigation, the numerical framework that was used guarantees stability, precision, & convergence over a broad spectrum of governing parameters.

To investigate the impact of key dimensionless attributes, a comprehensive parametric analysis was conducted. A variety of characteristics were measured, including those for MF strength, velocity slip, Pr&tl number, Grashof number, permeability, radiation, & bias. By methodically adjusting these parameters, one may have a solid underst&ing of how each of these aspects affects momentum & Heat exchange characteristics, both individually & in combination. Insights into optimization strategies & flow control are provided by this.

The data shows that the fluid's velocity is significantly reduced when an MF is present. This happens because the flow is opposed by the Lorentz force, a resistive force. As a result of deceleration, the thermal boundary layer thickens, which influences the

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surface-to-interior heat transfer rates. Such behaviour is crucial because MHD-based cooling systems control thermal performance using MF.

Additionally, the research demonstrates that velocity slip at the border reduces surface shear stress & has an effect on the velocity gradient in the vicinity of the wall. All throughout the investigation, this is shown. These effects are most noticeable in systems that are on the micro- & nano-scales, which are places where the usual continuum assumptions may no longer be acceptable. A further piece of evidence demonstrating that higher-order slip effects cannot be disregarded in rarefied flows & other technologically advanced applications is provided by the inclusion of first- & second-order slip conditions.

The sensitivity of flow & Heat exchange characteristics to ferromagnetic & viscoelastic processes was brought to light throughout the course of the numerical study that was conducted on stagnation point flow approaching a stretched surface. The examination of mixed convection flow over an inclined porous plate lays an emphasis on the combined influence of buoyancy forces, external flow, & the angle of inclination. This is similar to the way mixed convection flow is investigated. To be more specific, the inclination angle emerges as a very important parameter that influences both the processes of heat transmission & the processes of momentum transfer.

An overall validation of the accuracy & dependability of the computational technique that was chosen is provided by the fact that the current numerical findings are in close agreement with studies that have been published in the past. The modeling & numerical simulation of Newtonian fluid flow under complicated physical influences & border constraints is significantly advanced by this thesis, which offers a substantial contribution within the field. Consequently, the results have direct relevance to practical applications such as cooling technologies, energy systems, polymer processing, geothermal engineering, & magneto hydrodynamic devices, which has the effect of enhancing the research's scientific merit as well as its practical importance.

6.1 MAJOR FINDINGS

This study established that the flow behaviour of Newtonian fluids is strongly influenced by magnetohydrodynamic (MHD) effects. The application of a transverse

MF causes a noticeable flattening of velocity profiles because it introduces Lorentz forces that act against the fluid's motion. The magnitude of the magnetic damping effect grows in direct proportion to the values of the magnetic parameters. Therefore, engineering systems like electromagnetic pumps & cooling devices are ideal applications for MHD flow, which regulates fluid velocity. Because of velocity slip at the border surface, the flow properties are drastically changed close to the wall. When slip is introduced, as opposed to the conventional no-slip situation, surface shear stress is reduced & momentum border layer thickness is changed. The results show that Wall shear coefficients drop as the slip parameters rise, which has a detrimental effect on the fluid-surface interaction. Slip effects are crucial in many contexts, including micro-scale flows, rarefied gas dynamics, & nanofluidic applications. Under these conditions, this conduct takes on added importance.

Researchers have shown that heat & momentum transfer characteristics are significantly affected by the presence of a porous media. Greater permeability facilitates fluid passage through the porous structure, leading to thinner thermal border layers & larger velocity dispersion. Conversely, more resistance to flow causes lower velocities & higher temp as a consequence of less permeability. These studies provide light on the importance of porous media in thermal insulation, filtration systems, & geothermal applications.

Within the border layer, it would seem that thermal radiation is a substantial factor that has a large impact on the distributions of temp. In addition to increasing the density of the thermal border layer, increasing the radiation parameter would also result in an increase in the temp of the fluid. As a consequence of this, the pace at which heat is transmitted toward the surface is slowed down. In particular, this knowledge is beneficial to industrial operations that involve high temp. Some examples of such processes are solar collectors, nuclear reactors, & thermal control systems for spacecraft.

The study also shows that when external flow interacts with buoyancy forces, mixed convection effects significantly impact flow behaviour. The combined effects of thermal buoyancy & solutal buoyancy may raise or lower the fluid's velocity, depending on the variables that control its motion. The surface's angle acts as an extra regulator,

altering the fields of velocity & temp&, by extension, the Heat exchange efficiency.

Finally, numerical results show that upper-order slip conditions & instability effects cannot be ignored in sophisticated flow dynamics models. First- & second-order slip parameters heavily influence variations of the Nusselt number, Surface friction, & velocity gradients. It has been shown that the numerical methods used in this study display outstanding convergence & stability, thereby confirming their suitability for solving complex nonlinear border value problems in fluid mechanics.

6.2 FUTURE SCOPE OF THE STUDY

Although the present investigation offers substantial theoretical & numerical insights into Newtonian fluid flow with multiple physical effects, several directions remain open for further research. These future extensions can enhance both the depth & applicability of the current work.

- In order to more accurately depict complicated industrial fluids, polymer melts, & biological fluids, the current study might be expanded to include Nonlinear fluid model models such as Casson, Williamson, Carreau, Cross, or viscoelastic fluids.
- In order to further increase Heat exchange properties & investigate the synergistic effects of several Nano-objects, it is possible to investigate the integration of hybrid nanofluids or ternary nanofluids.
- There is the possibility that the research might be expanded to include flow configurations in three dimensions, which would result in a more accurate description of engineering & industrial systems that are used in practice.
- As part of future research, it may be possible to investigate the impacts of Chemical interactions in order to study reactive flows that are encountered in combustion processes, catalytic reactors, & applications of chemical engineering.
- In order to investigate temp-dependent Chemical interactions & mass transport processes, it is possible to add the impact of activation energy.

- When investigating the process of energy conversion from kinetic energy to internal energy, viscous dissipation effects may be included. This is especially true in flows that are both high-speed & high-viscosity.
- Incorporating Joule heating would lead to a better understanding of electrically conducting fluids that are exposed to intense MFs. This understanding would be applicable to MHD generators as well as plasma flows.
- For the purpose of improving the accuracy of the modeling, it is possible to take into consideration a variety of physical parameters, including temp-dependent viscosity, thermal conductivity, & diffusivity.
- In order to investigate the behavior of unsteady & transient flow, it is possible to conduct an analysis of the influence of time-dependent or oscillatory border conditions. There is the possibility that the investigation might be expanded to include a variety of geometrical configurations, including revolving disks, curved surfaces, shrinking sheets, or wavy borders.
- It is possible to do more research on slip border conditions of higher order in order to investigate rarefied gas flows & micro- & nano-scale transport phenomena.
- In order to broaden the range of applications, it is possible to include the effects of external forces, such as temp stratification, suction/injection, or Coriolis forces.
- More sophisticated numerical approaches, such as spectral methods, finite volume schemes, or adaptive mesh refinement, may be used in order to enhance the accuracy of the solution & the convergence of the numerical results.
- It is possible to make use of algorithms that are based on machine learning & artificial intelligence in order to make more accurate predictions about the behavior of flow & to optimize the parameters that regulate flow.
- In conclusion, it is possible to conduct experimental validation of the numerical findings in order to bolster the practical relevance of the theoretical models & to prove their application in engineering systems that are used in the real world.

6.3 FINAL REMARKS

The current thesis has been successful in developing a complete mathematical & numerical framework for understanding Newtonian fluid flow under the combined impact of multiple different physical processes that interact with one another. A more realistic & physically relevant explanation of border layer transport processes is provided by the research. This is accomplished by methodically including magnetohydrodynamic forces, velocity slip conditions, porous medium effects, thermal radiation, & mixed convection. The development of the governing equations & the subsequent numerical treatment of those equations constitute a major theoretical achievement in the science of fluid mechanics.

The addition of magnetohydrodynamic effects has made it feasible to perform an in-depth examination into the role of MFs in the control of fluid motion & the behaviour of Heat exchange characteristic. MFs may be employed as an effective control mechanism in electrically conducting fluids, as shown by the interaction In relation to the Lorentz force that is created & the flow of the fluid. This interaction is an example of how MFs Expression. This particular characteristic of the research is especially important to applications that include electromagnetic flow control, technologies that include cooling, & energy systems that are based on MHD.

A thorough investigation of the circumstances of velocity slip at the border has been carried out in order to overcome the constraints of the traditional assumption that no slip occurred. The findings make it abundantly evident that slip effects affect velocity profiles in a major way & lower surface shear stress, particularly in systems that operate at micro- & nano-scales. The research shows the significance of accounting for slip phenomena in modern technological applications & gives vital insight into the behavior of rarefied flow. This is accomplished by including higher-order slip conditions which are included into the study.

A thorough examination of the effects of porous media on thermal transport & fluid flow has shown that these media are critical in controlling the dispersion of momentum & temp. Porous structures are very effective at controlling temp because the permeability of the medium greatly affects fluid resistance & Heat exchange. In the

context of applications like geothermal systems, filtration processes, & thermal management devices, the ramifications of these findings are particularly important.

It has been shown that the impacts of thermal radiation may considerably alter the temp fields that are present inside the border layer, particularly in flow settings that are characterized by high temp. The findings of the numerical analysis show that radiative Heat exchange improves the transport of thermal energy & has a significant interaction with the other controlling factors. The significance of including radiation effects into the modeling of high-temp industrial & energy-related systems is highlighted by this fact.

The parametric analysis that was performed for a variety of flow configurations, such as stretched sheets, inclined porous plates, & stagnation point flows, provides a vivid illustration of the sensitivity of velocity, temp, & concentration fields to important physical factors. The systematic change of these parameters offers a more in-depth comprehension of the ways in which external forces, border conditions, & geometric factors may be efficiently regulated to alter the behavior of fluids.

It has been shown that computational methods are both effective & reliable in the resolution of highly nonlinear border layer problems, as demonstrated by the numerical solutions that were acquired during this overall inquiry. As a result of the fact that analytical solutions are either difficult to achieve or impossible to obtain in many instances, the numerical technique that was used in this research provides a practical & reliable alternative for the analysis of complicated flow systems.

The conclusions of this thesis, taken as a whole, provide a significant addition to the optimization of heat & momentum transfer processes in engineering systems. The mathematical models & numerical findings that were generated are directly relevant to a broad variety of practical sectors, such as cooling technologies, energy systems, polymer processing, & devices that are based on MHD. Consequently, this study not only contributes to the advancement of theoretical knowledge, but it also offers information that is very helpful for the design & development of contemporary applications involving Heat exchange& fluid flow.

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Numerical Approaches to the Mathematical Analysis of Physical Property Transfer in Newtonian Fluid Flow

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Abstract: The study of Newtonian fluid flow is fundamental in understanding numerous physical, engineering, and industrial processes. The transfer of physical properties such as momentum, heat, and mass within these fluids plays a critical role in fluid dynamics. This review focuses on the mathematical modeling of such transfer phenomena in Newtonian fluids using various numerical methods. The paper presents an overview of the governing equations, such as the Navier-Stokes equations and energy conservation principles, and emphasizes the significance of numerical techniques like Finite Difference Method (FDM), Finite Element Method (FEM), and Finite Volume Method (FVM) in solving these complex models. Particular attention is given to recent advances in computational fluid dynamics (CFD), stability analyses, and convergence studies that enhance the accuracy and efficiency of simulations. This review aims to provide a consolidated understanding of how numerical methods contribute to the precise prediction of physical property transport in Newtonian fluid systems.

Keywords: Newtonian Fluid, Physical Property Transfer, Numerical Methods, Finite Difference Method, Finite Element Method, Finite Volume Method, Computational Fluid Dynamics, Heat Transfer, Mass Transfer, Momentum Transport

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INTRODUCTION

The mathematical modeling of Newtonian fluid flow is a critical area of fluid dynamics with wide-ranging applications in engineering, chemical processing, environmental studies, and biological systems. Newtonian fluids, which exhibit a linear relationship between shear stress and strain rate, include commonly encountered substances such as water, air, and oil. Understanding how physical properties like momentum, heat, and mass are transferred within these fluids is essential for designing efficient systems and optimizing various industrial processes.

The complexities involved in fluid flow equations make analytical solutions difficult, especially when dealing with real-world boundary conditions and geometries. Numerical methods have emerged as indispensable tools for addressing these challenges. This review provides a detailed overview of the mathematical formulation of property transfer in Newtonian fluids and the numerical strategies employed to solve these problems.

MATHEMATICAL FRAMEWORK OF NEWTONIAN FLUID FLOW

Governing Equations

The behavior of Newtonian fluids is described by the **Navier-Stokes equations** for momentum conservation, along with the continuity equation for mass conservation and the energy equation for heat transfer. These can be summarized as:

- **Continuity Equation:**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

- **Momentum Equation:**

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \mu \nabla^2 \vec{v} + \vec{F}$$

- **Energy Equation:**

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Here, ρ is fluid density, $\vec{v}$ is velocity vector, p is pressure, μ is dynamic viscosity, T is temperature, and S is an energy source term.

MAGNETOHYDRODYNAMICS (MHD)

A magnetic field's effect on fluid flow may be seen in many contexts. The use of a magnetic field facilitates the processing of metals in liquid form. A thermonuclear fusion reactor uses a magnetic field to separate the heated plasma from the reactor's walls. Fluids that conduct electricity, such as electrolytes, plasma, liquid metals, and many more, are the focus of magnetohydrodynamics (MHD). The main idea behind MHD theory is that when a fluid is moving through a magnetic field, it creates an electric current. This current, in turn, causes the Lorentz force to act, altering the fluid's path. In 1832, the English physicist Michael Faraday conducted the first ever documented investigation on MHD. The electric current caused by the Thames River's velocity in Earth's magnetic field was something Faraday aimed to quantify. Unfortunately, the experiment did not succeed since galvanometers of that era had a very

limited capacity for detection. After Swedish electrical engineer Hannes Alfvén discovered Alfvén waves in 1942, MHD became well-known among scholars. According to Davidson (2016) and Hosking and Dewar (2016), Alfvén waves are a kind of wave-like oscillation that occur in magnetic field lines in a fluid that is very electrically conductive. In 1970, Alfvén was awarded the Nobel Prize in Physics for his tremendous and priceless contributions to MHD. Among the many fields that make use of MHD are those dealing with MHD power generators, magnetic drug targeting, magnetic endoscopy, nuclear reactor maintenance, solar flare and sunspot research, geomagnetic storm analysis, crude oil refineries, and MHD pump and brake design.

HALL CURRENT

American physicist Edwin Herbert Hall, while completing his doctorate at the esteemed Johns Hopkins University in Baltimore, Maryland, USA, in 1879, discovered the Hall effect. Within a transverse magnetic field, charges on a current-carrying conductor are redirected in an orthogonal direction to the electric current and the magnetic field. Therefore, the Hall current flows because a secondary potential difference is produced. When dealing with weak ionised gas densities or strong magnetic fields, the Hall effect becomes practically relevant.

NEWTONIAN FLUID

According to Newton's law of viscosity, a fluid that is considered Newtonian will have a stress that is directly proportional to the strain rate. Newtonian fluids include items like air, water, petrol, glycerol, and mercury, among others.

NON-NEWTONIAN FLUID

The stress-rate relationship in non-Newtonian fluids is nonlinear and depends on the strain rate. The Newtonian model does not apply to a wide variety of fluids used in industry and biology, including blood, gypsum paste, adhesives, lubricants, gels, paints, polymers, and synovial fluid. At a constant pressure and temperature, the apparent viscosity of a non-Newtonian fluid changes depending on the flow shape, strain rate, and deformation history (Chhabra and Richardson, 2008). Various non-Newtonian fluid types and Newtonian fluids are shown in Fig. 1 rheogram.

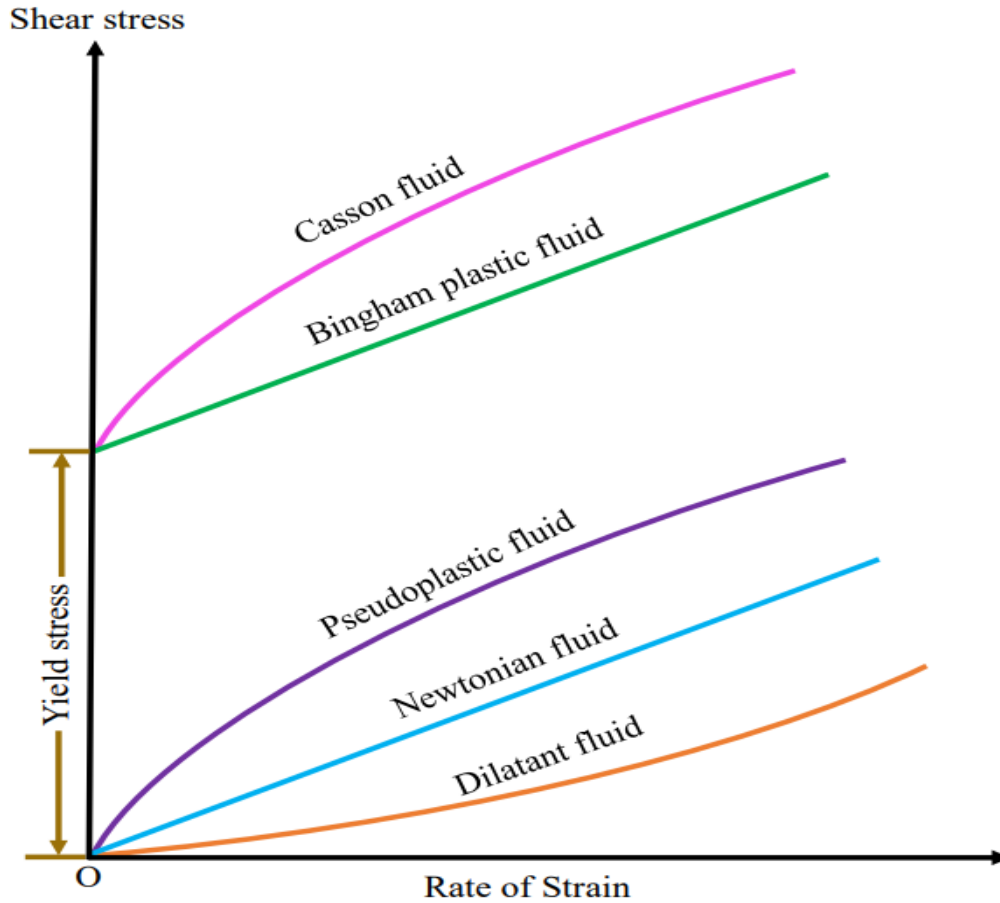


Figure 1: Rheogram of Newtonian and non-Newtonian fluids.

The apparent viscosity of pseudoplastic fluids, sometimes called shear-thinning fluids, decreases as the strain rate increases. Some fluids have shear-thinning properties; they include hair wash, paint, and ketchup. When it comes to apparent viscosity, shear thickening/dilatant fluids show the inverse pattern. Instances of shear-thickening fluids include cornstarch-water mixtures, wet sand, and china clay suspensions. For a Bingham plastic fluid to behave like a Newtonian fluid, the applied shear stress must be greater than a certain threshold value, called the yield stress. The rheological properties of non-Newtonian fluids are investigated in this thesis using the Casson fluid, second-grade fluid, and Walters B' fluid models circulation of fluids.

HEAT TRANSFER

When there is a difference in temperature between two objects, this is called heat transfer. The three main processes that cause this heat exchange are radiation, convection, and conduction, as shown in Figure 2. Intermolecular interactions allow for the passage of heat from a substance's more energetic molecules to its less energetic ones; this process is known as conduction.

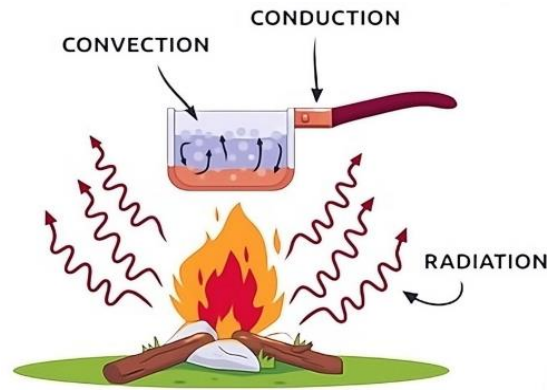


Figure 2: Modes of heat transfer.

MASS TRANSFER

Mass transfer describes the movement of stuff from a densely populated area to an open one. When an alien species is present in a fluid, it creates a concentration gradient that makes mass transfer easier. One example is the colour shift that occurs when a little quantity of potassium permanganate is added to water, turning it purple. The scent of perfume wafts around the room when the wearer applies it. Another example of mass transfer is the process of dissolving sugar in lemonade, which requires a teaspoon. Absorption of nutrients into the circulation, blood oxygenation, ion movement via osmosis, water purification, distillation, and many other biological and chemical processes rely on mass transfer. Both diffusion and convection play a role in the movement of mass through fluids. The species shown in Fig. 3 undergo diffusive mass transfer as a result of molecular mobility.

The transmission of physical attributes such as momentum, heat, and mass in Newtonian fluid flow involves substantial mathematical and computational hurdles. The Navier-Stokes equations in particular are notoriously difficult to solve analytically, especially when dealing with complex boundary conditions and geometries, despite the fact that Newtonian fluids are simplified by the assumption of constant viscosity. When applied to real-world scenarios, traditional analytical approaches fail to provide adequate solutions. Consequently, trustworthy numerical approaches must be used and developed to adequately manage these intricacies and provide precise forecasts of fluid behaviour and property transfer.

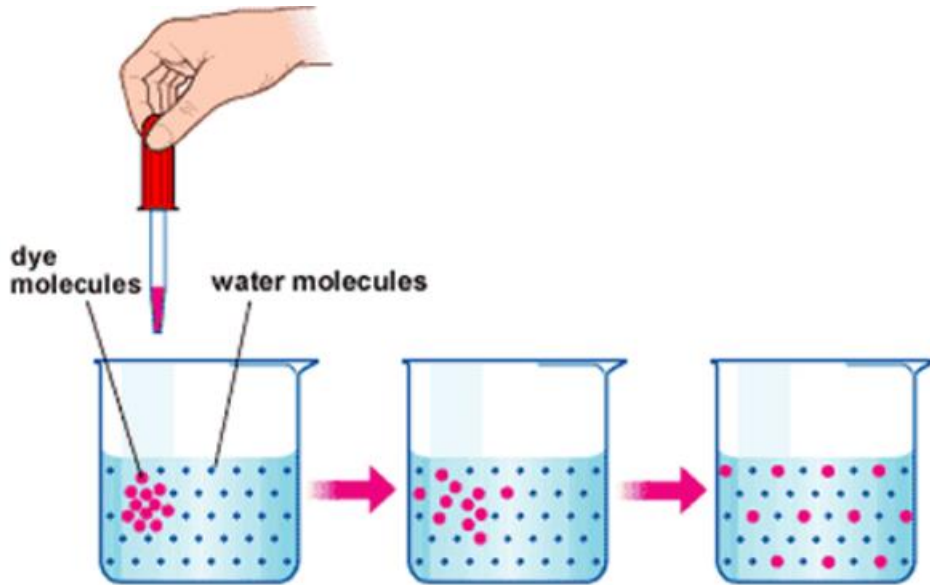


Figure 3: Propagation of dye molecules by diffusion

Conversely, convective mass transfer describes the situation where the species spreads together with the fluid's bulk flow. The glucose produced by plants during photosynthesis is transported to different sections of the plant by the water absorbed by the soil. Because of its relevance in many fields, including environmental engineering, electrical engineering, metallurgy, refrigeration, pharmaceutical drug development, chemical constituent separation in distillation apparatus, HVAC system design, and automobile engineering, the study of MHD flow with heat and mass transfer has captivated many researchers.

CROSS-DIFFUSION EFFECTS

It is common for a concentration gradient to cause mass diffusion and a temperature gradient to cause heat diffusion. Mass diffusion occurs as a result of temperature gradients and vice versa when there are high concentration and temperature gradients (Kafoussias and Williams, 1995). The Soret effect is defined as the mass flow that results from a temperature gradient, while the Dufour effect is defined as the heat flux that results from a concentration gradient.

Swiss physicist Charles Soret discovered the Soret effect in 1879. Two tubes containing sodium chloride (NaCl) and potassium nitrate (KNO₃) solutions were placed in front of him for experimental observation. The tubes were either straight or U-shaped. Both tubes had one end in a bath of icy water and the other in a pot of boiling water. The experiment proved the existence of the Soret effect by showing that the solute settled to the bottom of the tubes as they were chilled. Species migrate from warm to cold areas as a result of the positive Soret effect, and vice versa as a result of the negative Soret effect (Platten, 2006). A favourable Soret effect is more often seen in heavier species, whereas a negative Soret impact is more

commonly seen in lighter species. In 1873, the Dufour effect was discovered by a Swiss scientist called L. Dufour. As an example, the Dufour effect would cause a temperature increase of several degrees in a combination of hydrogen and nitrogen, which are held at various concentrations. Rowley and Horne (1980) conducted experimental studies on the Dufour effect in a combination of cyclohexane and carbon tetrachloride. Gaseous mixes of intermediate molecular mass (Nitrogen and air) exhibit the Dufour effect, according to Eckert and Drake (1972), but combinations of gases of little molecular mass (Hydrogen and helium) exhibit the Soret effect. Isotope separation, fuel cell design, nuclear reactor engineering, petrology, and gas impurity removal are just a few of the many engineering fields that make use of the Soret-Dufour phenomena, which represent a cross-diffusive coupled heat and mass transfer in fluid flow.

CHEMICAL REACTION AND ACTIVATION BENERGY

In a chemical reaction, bonds in the reactant species are broken, allowing bonds in the product species to be formed. The concentration of the species reacting determines the pace of the chemical reaction. When the concentration of the reactant species is directly proportional to the first power of the reaction rate constant, we say that the reaction is of first order.

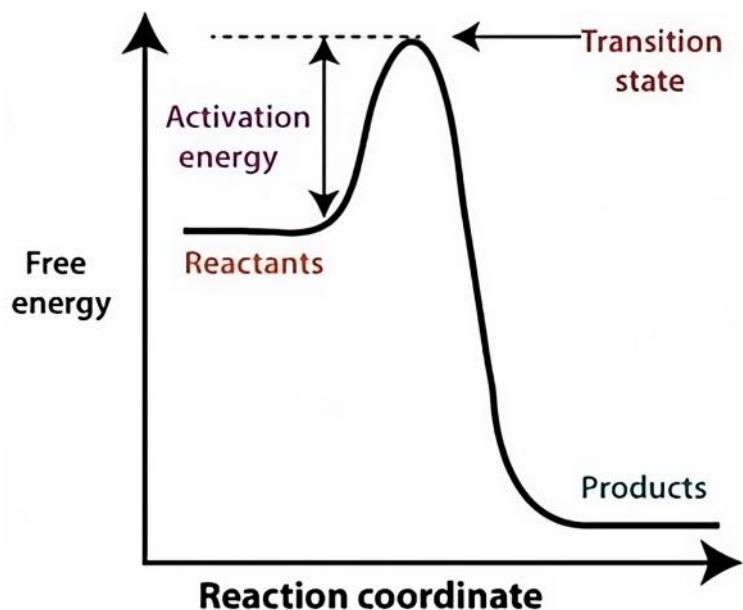


Figure 4: Activation energy.

There is a tight relationship between the activation energy and the reaction rate in chemistry. Chemical reactions may only take place when the reactants reach a certain threshold energy, according to Swedish scientist Svante Arrhenius's 1889 discovery. It was Arrhenius who first used the word "activation energy" to describe the bare minimum of energy that molecules of

the reactants needed to start a chemical reaction. In order for the reactants to collide and start forming product species, they must overcome this energy gap, as depicted in Fig. 4.

The rate constant, activation energy, and temperature are all variables in the Arrhenius equation, which, as stated by Tencer et al. (2004), is:

$$k_c = A \exp \left(-\frac{E_a}{k_B T} \right),$$

Where A is the Arrhenius pre-exponential factor, E_a is the activation energy, and $k_B = 8.61 \times 10^{-5}$ eV/K is the Boltzmann constant. Numerous fields, including chemical engineering, geothermal engineering, food processing, oil emulsions, polymer manufacture, and ceramic or glassware production, benefit from studying MHD flow in relation to chemical reaction and activation energy.

POROUS MEDIUM

The solid matrix of a porous material has interconnected spaces called pores. As shown in Figure 5(a), the fluid is able to move across the medium thanks to the web-like network formed by the linked pores. Porous media include things like sand, wood, limestone, furnace refractory material, sponges, fabric, fibreglass, ceramics, and even human lungs. As shown in Figure 5(b), pumice is a very porous igneous rock that forms when lava is quickly cooled and depressurised after being expelled during a volcanic eruption. To measure porosity, take the volume of the pores and divide it by the volume of the medium. The porosity value for naturally porous medium is less than or equal to 0.6 (Nield and Bejan, 2017).

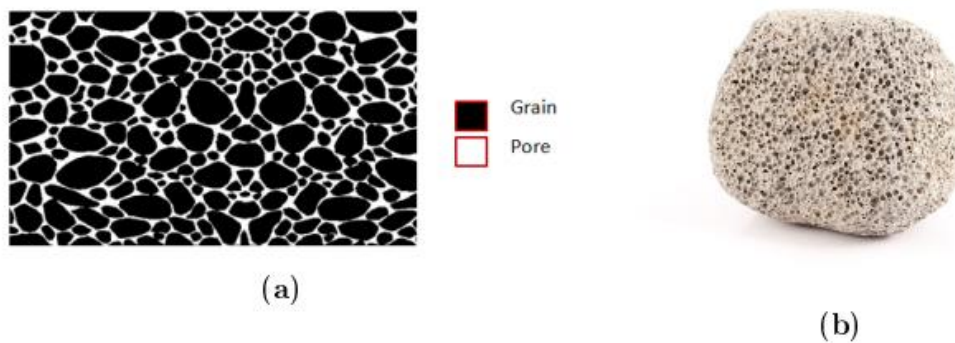


Figure 5: (a) Porous medium and (b) Pumice.

Most flows occurring below ground have Reynolds numbers that fall within the Darcy's range. On the other hand, in cases with steep gradients or with big solid particles in the flow, the fluid's trajectory takes a curved shape, which causes it to experience inertial acceleration. It follows that a non-Darcy equation must be used, taking into account both the inertia caused by the high-velocity flow and the typical stress caused by the distortion velocity. As a result,

a number of models that do not include Darcy have been developed. The Darcy-Forchheimer law, which was proposed by Forchheimer (1901) and is expressed as the quadratic form-drag in Darcy's law (Eq), is:

$$\nabla p = -\frac{\mu}{k_p} \vec{V} - \frac{\rho C_F}{\sqrt{k_p}} |\vec{V}| \vec{V},$$

Where C_F is the non-dimensional form-drag coefficient. The study of convective flow in porous media is useful in many different contexts, including but not limited to: groundwater hydrology, magnetic-levitation casting, geothermal reservoirs, decontamination processes, alloy solidification, microporous heat sinks, blast furnaces, and catalytic converters.

NANOFLUID AND HYBRID NANOFLUID

The use of effective coolants is essential for the upkeep of complex machinery and industrial equipment. Power generation, chemical manufacture, food processing, automobile engines, nuclear reactors, textile production, and microelectronics are just a few examples of industries that have a significant need for effective cooling techniques. The rate of heat transfer between the solid surface and the fluid also has a significant impact on the produced product's quality. Metals, oxides, and carbon nanotubes (CNTs) often have a much higher thermal conductivity than more traditional base fluids such as water (H₂O), engine oil (EO), and ethylene glycol (EG). Recent developments in nanotechnology have made it possible to enhance the base fluid's heat transmission properties by adding nanoparticles. One kind of nanoparticle is colloidally suspended in the base fluid of a nanofluid, making it an increased heat transfer fluid. The groundbreaking study on the improvement of heat transmission in Cu – H₂O nanofluid was conducted by Choi and Eastman (1995), who also established the term "nanofluid". The single-phase and double-phase models are the main tools for studying nanofluid flow. According to the single-phase model, the base fluid and nanoparticles move at the same speed because of thermal equilibrium. According to Albojamal and Vafai (2017), the nanofluid's heat transfer properties may be altered by focussing on the two phases of the model: the solid nanoparticle phase and the fluid phase. The rate of heat transfer in the cavity flow of Cu – H₂O nanofluid was investigated by Tiwari and Das (2007) using the single phase model. As a function of the volume percentage of nanoparticles, the authors have given a detailed list of the thermo-physical characteristics of nanofluid.

Researchers have created a new kind of fluid called a hybrid nanofluid, which has several kinds of nanoparticles mixed with the base fluid, to improve the thermo physical characteristics of nanofluids. For the thermo-physical characteristics of hybrid nanofluid, the

correlations have been supplied by Devi and Devi (2016). Due to their diverse applications in nano-drug administration, heat exchangers, refrigeration, solar collectors, supercomputers, autos, cryosurgery, cancer treatment, transformers, heat pumps, photoelectric equipment, textile industries, geothermal power extraction, and nuclear reactor maintenance, nanofluid and hybrid nanofluid flow research has recently attracted the interest of many researchers.

CATTANEO-CHRISTOV DOUBLE DIFFUSION MODEL

The Fourier law expresses the heat flux ($\vec{q}$) as:

$$\vec{q} = -\kappa \nabla T,$$

Whereas the mass flux ($\vec{j}$) is represented by the Fick's law as:

$$\vec{j} = -D_m \nabla C.$$

The energy equation becomes parabolic when Fourier's heat flux and Fick's mass flow are both included, and the species concentration equation becomes parabolic as well. This means that an initial disturbance is instantly distributed over the whole domain. In other words, the "paradox of heat conduction" holds true, and the heat disturbance travels at an unlimited pace. Cattaneo (2019) offered the Maxwell-Cattaneo (MC) law as a solution to this unrealistic occurrence; this law incorporates the thermal relaxation time component into the Fourier's heat flow and is expressed as:

$$\vec{q} + \lambda_T \frac{\partial \vec{q}}{\partial t} = -\kappa \nabla T,$$

Where λ_T is the thermal relaxation time, which signifies the time lag for acquiring steady-state heat conduction in a fluid element exposed to a sudden temperature gradient. Most materials have a thermal relaxation time that is negligible, measured in picoseconds at the most. As to Chandrasekharaiah (1998), components such as sand (21s), glass ballotini (11s), ion exchanger (54s), H⁺ acid (25s), and NaHCO₃ (29s) cannot be disregarded.

By using the MC rule, the energy equation takes on a hyperbolic form, suggesting that heat disturbances travel at a limited wave speed. Christov (2009) made an additional adjustment by substituting the Oldroyd upper convected derivative for the partial time derivative; this latter function generalises the MC rule to a frame-independent extent. The concurrent heat and mass diffusion represented via the Cattaneo-Christov theory is termed the Cattaneo-Christov double diffusion model, and the respective expressions for $\vec{q}$ and $\vec{j}$ are (Shankar and Naduvinamani, 2019):

$$\vec{q} + \lambda_T \left(\frac{\partial \vec{q}}{\partial t} + (\vec{V} \cdot \nabla) \vec{q} - \vec{q} \cdot (\nabla \vec{V}) + (\nabla \cdot \vec{V}) \vec{q} \right) = -\kappa \nabla T,$$

$$\vec{j} + \lambda_C \left(\frac{\partial \vec{j}}{\partial t} + (\vec{V} \cdot \nabla) \vec{j} - \vec{j} \cdot (\nabla \vec{V}) + (\nabla \cdot \vec{V}) \vec{j} \right) = -D_m \nabla C,$$

Where λ_C denotes the mass relaxation time. According to Rehman et al. (2023), the Cattaneo-Christov double diffusion scheme accurately represents the heat and mass transport processes caused by concentrated and high-temperature gradients as well as by fields of changing concentration and temperature.

ENTROPY GENERATION

Entropy is created during an irreversible process, according to the second rule of thermodynamics. Every time energy is transformed into usable work, some of that energy is lost, reducing the system's efficiency. Because of the inherent irreversibilities in thermodynamic processes, entropy formation occurs in direct proportion to this energy deterioration. Entropy production in convective heat transmission was the subject of Bejan's seminal research (1979). The analysis takes into account the effects of conduction heat transfer and viscous dissipation on the entropy development rate. When heat and mass transmission are occurring at the same time, Mourad et al. (2006) examined how entropy is generated. So far as we can tell from Mourad et al. (2006), the SG rate at the local level is:

$$\begin{aligned} S_G = & \frac{\kappa}{T^2} \left(\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right) \\ & + \frac{\mu}{T} \left(2 \left(\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right) \\ & + \frac{RD_m}{C} \left(\left(\frac{\partial C}{\partial x} \right)^2 + \left(\frac{\partial C}{\partial y} \right)^2 \right) \\ & + \frac{RD_m}{T} \left(\left(\frac{\partial T}{\partial x} \right) \left(\frac{\partial C}{\partial x} \right) + \left(\frac{\partial T}{\partial y} \right) \left(\frac{\partial C}{\partial y} \right) \right), \end{aligned}$$

In where R stands for the ideal gas constant. Here, heat transfer by conduction (first term), viscous dissipation (second term), diffusive mass transfer due to concentration gradient alone (third term), and cross-diffusive mass transfer due to temperature and concentration gradients (fourth term) all contribute to entropy generation. Improving the system's performance relies heavily on analysing entropy creation, which aids in reducing the elements that degrade energy. Numerous fields rely on entropy analysis, including biological systems, electronic

cooling, geothermal energy generation, solar collectors, magnetic refrigeration, and solid-state physics.

BOUNDARY LAYER THEORY

German physicist Ludwig Prandtl proposed the boundary layer hypothesis in 1904. Because there is no frictional loss when a thin layer of a fluid with a low viscosity runs over a solid surface that is not moving, the flowing fluid will stick to the solid. Subsequent fluid layers are retarded by this stationary layer, and the process continues thereafter in the same manner. But as we go further away from the bounding surface in a normal direction, the retarding effect on the fluid layers decreases. Consequently, a thin layer known as the boundary layer forms close to the boundary surface, causing the fluid velocity to increase from zero at the boundary surface and eventually approach the free stream velocity ($U_\infty(x)$).

Rotation and viscous forces have a major impact inside the boundary layer. The outer layer is the area outside the boundary layer where the flow is inviscid and irrotational. A high Reynolds number (Re) is required for a boundary layer to form in a fluid. Since the viscous forces would be noticeable over the whole flow domain if Re is minimal, the boundary layer would not be present. The boundary layers for concentration and heat are described similarly when mass and heat transfer are taking place at the boundary. Unlike in the situation of inviscid Euler equations, boundary layer theory allows one to determine the viscous drag force by specifying the no-slip condition at the solid border. In addition, by analysing the components in terms of order of magnitude, the unnecessary ones may be removed from the boundary layer flow Navier-Stokes equations, making them simpler. Schlichting and Gersten provide a comprehensive analysis of boundary layer theory and its practical applications (2017). For a Newtonian fluid in two dimensions, the Prandtl boundary layer equations for a steady incompressible flow without body forces are (Bansal, 1977):

SIMILARITY TRANSFORMATION

In a two-dimensional boundary layer flow with velocity field $\vec{V} = (u(x, y), v(x, y))$, the non-dimensional velocity component $u/U_\infty(x)$ varies with only one variable $\eta =$

$y/g(x)$, where $g(x)$ is proportional to the boundary layer thickness ($\delta(x)$), and η is called the similarity variable (Som, et al., 2012; Bansal, 1977; Kundu and Cohen, 2004). This change, known as the similarity transformation, converts the dimensional system of PDEs into a non-dimensional system of ODEs that are conceptually identical. German physicist Heinrich Blasius first proposed the similarity transformation in 1908 as a solution to the boundary layer

flow issue across a flat plate. A Newtonian fluid's continuous incompressible flow across a semi-infinite flat plate with a constant free stream velocity (U_∞) was something that Blasius took into consideration.

SUCTION/INJECTION

To manage the boundary layer's development, suction/injection applications are critical. To keep the boundary layers from separating and to increase the lift coefficient, suction is an essential tool in aerodynamic engineering. The boundary layer flow, which counteracts the negative pressure gradient close to the bordering surface, is drawn in by the slowed fluid molecules via suction. To reduce the likelihood of boundary layer separation, fast-moving fluid molecules counteract the slow-moving ones. In addition, the flow separation is hindered by the kinetic energy delivered by the tangential injection of fluid into the boundary layer. The impact of suction on an airfoil's aerodynamic lift coefficient was studied by Shoejaefard et al. (2005). The findings show that injection lowers the skin friction coefficient, whereas suction increases the lift coefficient. Spacecraft re-entry, rocket engine propellants, ablative and heat sink cooling, and transpiration cooling the process of sucking or injecting fluid through porous surfaces have industrial and technical uses.

DIMENSIONAL ANALYSIS

Researching flow behaviour on a full-scale model is a laborious, costly, and complex process. For this reason, researchers often choose to test the prototype's flow behaviour on a smaller scale before making any firm predictions about the prototype's actual performance. The non-dimensional factors ensure that the model flow and the prototypical flow are identical (White, 2011; Cengel and Cimbala, 2014). It is common for fluid dynamics issues to have four main dimensions: mass (M), length (L), time (t), and temperature (T). These dimensions are used to construct the secondary dimensions. The dynamic viscosity dimension, for instance, is given by $[M L^{-1} t^{-1}]$. An essential tool for non-dimensionalizing the governing equations is dimensional analysis. By combining the issue's dimensional parameters into a collection of non-dimensional parameters, we may simplify the problem and do fewer trials. Adopting the Buckingham Pi theorem, established by Buckingham (1914), yields the non-dimensional parameters. Dimensional analysis is useful for a number of reasons, including deciphering experimental results, tackling issues that are too complex for straightforward techniques to handle mathematically, and determining the relevance of a particular physical occurrence.

BOUSSINESQ APPROXIMATION

The buoyant force components in the equations that regulate natural convective flow are approximated using the Boussinesq method. According to Incropera et al. (2006), the momentum equation is the only one that takes into account the fluctuation of fluid density in incompressible natural convective flows.

BOUNDARY CONDITIONS

Boundary conditions are set at the edge of the flow domain and are used to solve the system of differential equations regulating the flow. Common boundary conditions seen in fluid dynamics include Dirichlet, Neumann, and Robin's. At $y = 0$, the circumstances at the solid surface are given, whereas at $y \rightarrow \infty$, the ambient conditions are given.

NUMERICAL METHODS FOR NEWTONIAN FLUID FLOW

Finite Difference Method (FDM)

The finite difference method discretizes the partial differential equations using approximate difference equations. It is commonly applied to simple geometries and is widely used due to its straightforward implementation.

Finite Element Method (FEM)

The finite element method provides flexibility in handling complex geometries and boundary conditions. It divides the fluid domain into smaller elements and applies variational principles to derive discrete equations.

Finite Volume Method (FVM)

The finite volume method is highly popular in computational fluid dynamics. It ensures conservation of mass, momentum, and energy within each discrete control volume, making it suitable for both structured and unstructured grids.

HEAT AND MASS TRANSFER IN NEWTONIAN FLUIDS

The simultaneous transfer of heat and mass is essential in processes like chemical reactors, heat exchangers, and environmental flows. Numerical models often incorporate convection-diffusion equations to simulate these phenomena. Studies show that coupling momentum, heat, and mass transfer equations improves the accuracy of predictions in mixed convective flows and phase change problems.

ADVANCES IN COMPUTATIONAL FLUID DYNAMICS (CFD)

The development of CFD software and high-performance computing has significantly expanded the capability of numerical fluid analysis. Techniques such as adaptive meshing, turbulence modeling, and parallel computation are now routinely used to solve large-scale Newtonian fluid flow problems with enhanced precision.

STABILITY AND CONVERGENCE ANALYSIS

Ensuring the numerical stability and convergence of the solutions is critical. Techniques like the von Neumann stability analysis and grid independence studies are essential to validate numerical models. Improved time-stepping schemes and iterative solvers have also contributed to more stable and accurate solutions.

CONCLUSION

The mathematical study of physical property transfer in Newtonian fluids is essential for advancing engineering solutions and optimizing industrial processes. Numerical methods such as FDM, FEM, and FVM have revolutionized this field by providing accurate and computationally efficient tools to solve complex flow problems. With the continued development of computational resources and numerical algorithms, future studies are expected to offer even more precise insights into the dynamics of Newtonian fluids across various applications.

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CERTIFICATE

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An Numerical Study of Magnetized Mixed Convection Nanofluid Flow Over A Curved Surface

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Abstract: Motivated by its significance for advanced thermal and commercial applications, this work offers a computational analysis of magnetized mixed convection nanofluid flow across a curved stretched surface. Using the Tiwari–Das single-phase nanofluid model, the study takes into account the effects of a transverse magnetic field, curvature, mixed convection, and important thermophysical factors. Through appropriate similarity transformations, the controlling nonlinear partial differential equations are converted into a system of dimensionless ordinary differential equations, which are then numerically solved using the shooting approach in conjunction with a fourth-order Runge–Kutta scheme. A detailed analysis is conducted of the effects of different factors on the local Nusselt number, skin friction coefficient, and velocity and temperature profiles. The findings provide important new information for the design and optimization of curved surface thermal systems using nanofluids by demonstrating how magnetic and curvature effects dramatically change flow behavior and heat transmission properties.

Keywords: Nanofluid Flow, Magnetized, Numerical Study, Curved Surface, Runge–Kutta.

INTRODUCTION

Mixed convection boundary layer flow over curved surfaces has received considerable attention in recent years due to its importance in many engineering and industrial processes such as thermal energy systems, aerospace components, biomedical devices, and electronic cooling technologies. Unlike flat surfaces, curved geometries significantly influence the momentum and heat transfer characteristics of the flow. When forced convection induced by surface stretching interacts with buoyancy-driven natural convection, the resulting mixed convection flow exhibits complex physical behavior that demands rigorous numerical investigation. [1][2]

Nanofluids have emerged as a viable method for improving the efficiency of heat transmission in thermal systems, and this approach has become more popular. When tiny particles are suspended in traditional base fluids, nanofluids are produced. These nanofluids have increased thermal conductivity and heat transmission capacities. Due to the fact that it is straightforward and trustworthy in terms of forecasting thermal behavior, the Tiwari–Das single-phase nanofluid model has been extensively embraced among the several theoretical models that are now accessible. [3][4] Nanofluids have been shown to dramatically improve heat transfer rates in stretched and curving surface flows under a variety of different physical situations, as evidenced by a large number of investigations. [5][6]

Magnetohydrodynamic (MHD) effects are very important in the field of electrically conducting nanofluid flows. This is particularly true in applications such as the cooling of nuclear reactors, metallurgical processes, magnetic drug delivery, and microfluidic devices. [7] The use of a transverse magnetic field results in the production of Lorentz forces that are in opposition to the motion of the fluid, which in turn causes the distributions of velocity and temperature to be altered. There have been several experiments that have found that magnetic fields reduce velocity profiles while simultaneously increasing the thickness of the thermal boundary layer [8][9] The majority of these studies, on the other hand, are restricted to flat geometries and do not take into account the simultaneous impact of curvature and mixed convection effects.

Inspired by these findings, the current research focuses on simulating the flow of a nanofluid subjected to magnetized mixed convection across a curved stretching surface. The detailed examination of the impacts on velocity, temperature, skin friction coefficient, and heat transfer rate is carried out by major controlling factors such as the magnetic parameter, mixed convection parameter, curvature parameter, Prandtl number, and nanoparticle volume percent. [10] A dimensionless form is created for the governing nonlinear equations, and then they are numerically solved. The results of this research help improve thermal systems that use nanofluids and shed light on how to maximize heat transfer efficiency in configurations with curved surfaces. [11][12]

OBJECTIVES

- To study the effects of magnetic field and mixed convection on nanofluid flow over a curved surface.
- To examine heat, transfer and skin friction characteristics under varying physical parameters.

RESEARCH METHODOLOGY

Physical Model and Flow Assumptions

A continuous, two-dimensional, incompressible mixed convection nanofluid flow across a curved stretching surface of constant radius R is considered. The surface expands linearly, forcing convection, while temperature differences between the surface and ambient fluid cause buoyancy forces and natural convection. Assuming the nanofluid is electrically conducting, a uniform transverse magnetic field of intensity B_0 generates Lorentz forces that oppose flow. The velocity and temperature boundary layers along the curved geometry are greatly affected by forced and free convection. Many common assumptions are used for mathematical tractability. The boundary layer approximation and single-phase Tiwari–Das formulation simulate the nanofluid, assuming thermal equilibrium between the base fluid and nanoparticles. By assuming a small magnetic Reynolds number, the generated magnetic field may be ignored. Since viscous dissipation and Joule heating are negligible at moderate flow conditions, they are neglected. In the Boussinesq approximation, density fluctuations are only addressed in the buoyancy term, while all other nanofluid thermo-physical parameters remain constant.

Governing Equations

Under the above assumptions, the governing equations for continuity, momentum, and energy in curvilinear coordinates (x, y) are:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_{nf}} (u - U_\infty) + g \beta_{nf} (T - T_\infty)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2}$$

Where u, v are velocity components, ν_{nf} is nanofluid kinematic viscosity, and α_{nf} is thermal diffusivity.

Similarity Transformation

To convert the governing PDEs into ODEs, the following similarity transformations are introduced:

$$\eta = \sqrt{\frac{a}{\nu_f}} y, \quad \psi = \sqrt{a \nu_f} x f(\eta)$$

$$u = a x f'(\eta), \quad v = -\sqrt{a \nu_f} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Substitution yields the dimensionless momentum and energy equations:

Momentum equation

$$f''' + f f'' - (f')^2 - M(f' - 1) + \lambda \theta = 0$$

Energy equation

$$\theta'' + Pr f \theta' = 0$$

Where

- $M = \frac{\sigma B_0^2}{\rho_f \alpha}$ is the magnetic parameter.
- $\lambda = \frac{Gr}{Re^2}$ is the mixed convection parameter.
- Pr is the Prandtl number.

Boundary Conditions

The transformed boundary conditions are:

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1$$

$$f'(\infty) \rightarrow 1, \quad \theta(\infty) \rightarrow 0$$

These conditions represent no-slip and prescribed temperature at the curved surface, while free-stream conditions prevail far from the surface.

Numerical Solution Procedure

The shooting approach in conjunction with a fourth-order Runge–Kutta scheme is used to numerically solve the resultant nonlinear coupled ODEs. The Newton-Raphson approach is used to iteratively estimate the missing beginning conditions until convergence is reached with an accuracy of 10^{-6} .

The physical quantities of engineering interest are computed as:

Skin friction coefficient

$$C_f = \frac{\tau_w}{\rho U_\infty^2} = f''(0)$$

Local Nusselt number

$$Nu_x = -\theta'(0)$$

RESULTS

The system of nonlinear ordinary differential equations is obtained by changing the differential equations and boundary conditions. First changing boundary value problems into initial value problems and then using the finite difference approach, we solved most of them quantitatively. Step size is assumed to be 0.01. To meet asymptotic convergence criteria, we choose η (max) = 15 and estimate an error tolerance of 105. To meet far-field boundary criteria, trial values of $\varsigma'''(0)$, $f''(0)$, $\varepsilon'(0)$, and $\phi'(0)$ were modified repeatedly. Ferrofluid viscosity increases and velocity distribution reduces as ferromagnetic parameter increases. Because ferrofluid has microparticles. The temperature profile rises sharply. Because ferrofluid particles interact with the magnetization effect, this event occurs. The $f'(\eta)$ has a declining trend, but the $\theta(\eta)$ temperature profile has the opposite effect.

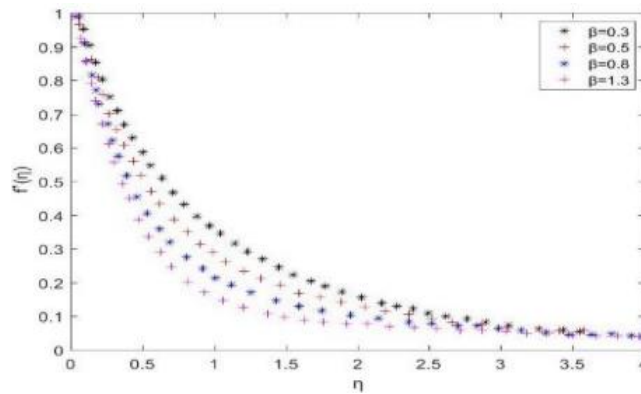


Figure 1: $f'(\eta)$ versus β

Figure displays the velocity profile variation $f'(\eta)$. Variation in ferromagnetic interaction parameter β across velocity profiles is observed. Observation reveals a significant decrease in velocity dispersion as β grows. The ferrofluid's ferromagnetic particles interact strongly with the magnetic field, causing this phenomenon. As β increases, magnetic forces on the fluid intensify, generating a resistive Lorentz force that opposes fluid movement. This increases the ferrofluid's effective viscosity, reducing momentum transmission in the boundary layer. Due to this resistance, fluid velocity reduces near and distant from the surface, reducing momentum barrier layer thickness. Since the velocity profile has dropped, ferromagnetic contact seems to be the main factor affecting flow. Such control is useful in engineering systems that demand flow suppression or stability. Typical systems include magnetic drug targeting, cooling, and ferrofluid actuators. The study confirms that β significantly affects the hydrodynamic performance of ferrofluid flow.

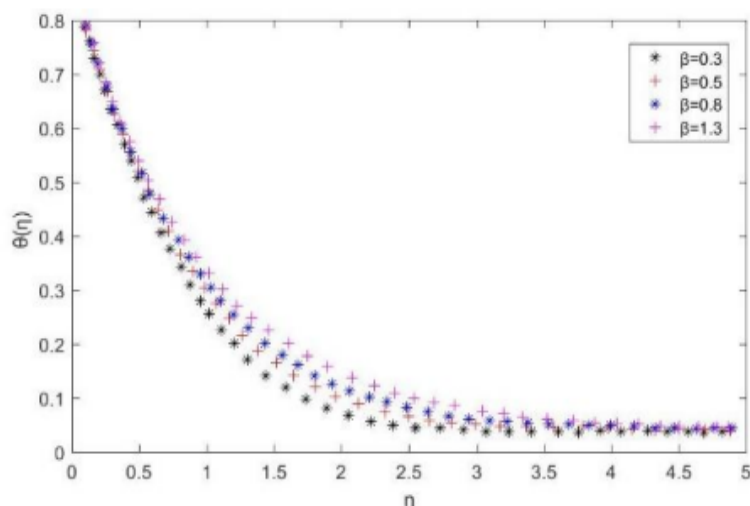


Figure 2: (η) versus β

Figure shows that the viscoelastic parameter (N) determines fluid velocity. As N grows, velocity profile quality improves. As seen in the graph, increasing the viscoelastic parameter N decreases fluid mobility near the stretched sheet but increases it further away. Heat transmission decreases as N increases, allowing fluid to flow faster. As N increases, the dimensional stream function and velocity expand. Figure 3.5 shows that viscoelasticity in fluids causes a temperature distribution change proportional to η . The temperature profile decreases as the viscoelastic parameter rises.

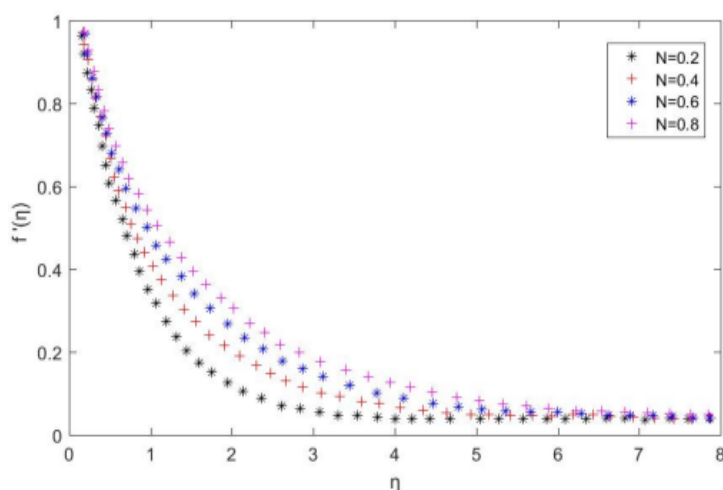


Figure 3: $f'(\eta)$ versus N

The velocity profile $f'(\eta)$ fluctuates as illustrated in Figure. With different viscoelastic parameter N values, this variance occurs. As N grows, the boundary layer velocity profile rises. Elastic viscoelastic fluids store mechanical energy and release it to aid fluid motion,

which may explain this phenomenon. This optimizes smooth movement. Due to elastic resistance, viscoelastic effects near the stretched surface reduce mobility somewhat. However, identical effects boost momentum transfer when distant from the surface. Increasing N makes the fluid more elastic, improving the velocity distribution. This improvement shows that viscoelasticity accelerates flow, especially distant from the surface. This characteristic is useful in viscoelastic fluid applications such polymer processing, coating flows, and medicinal fluids.

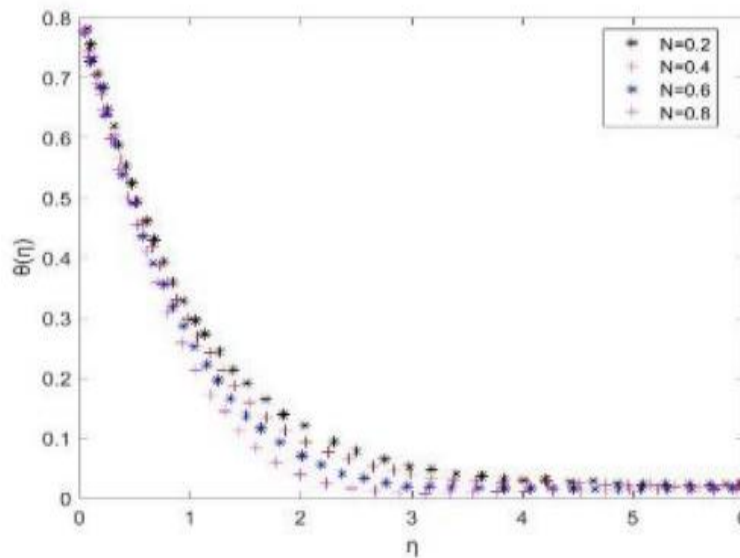


Figure 4: (η) versus N

The viscoelastic parameter N significantly affects the temperature profile $\theta(\eta)$, as seen in Figure. When nitrogen concentrations rise, temperatures fall. Viscoelastic processes reduce heat transport in the fluid, resulting in a smaller thermal boundary layer. Because of its elastic properties, the fluid retains less thermal energy, improving boundary heat dissipation. The decline in temperature profile shows that greater viscoelastic parameter values enhance heat transmission efficiency. These results suggest that viscoelastic fluids may be employed in thermally controlled applications. Polymeric fluid cooling systems and heat exchangers are examples.

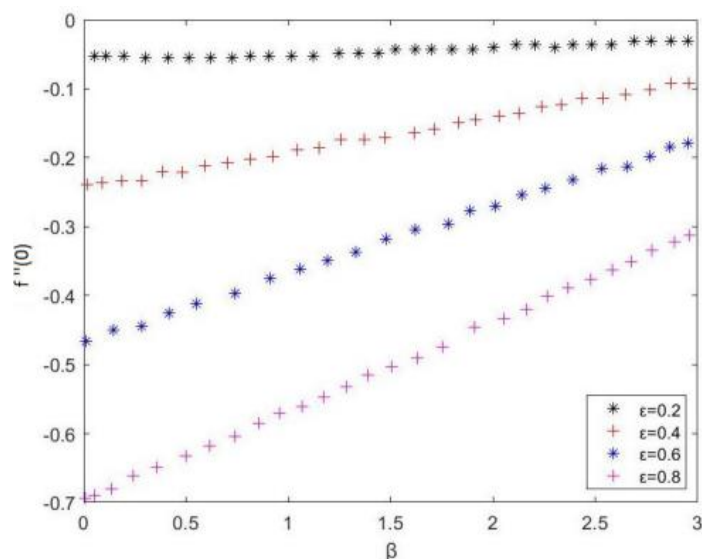


Figure 5: Skin Friction Coefficient versus variable E

Figure illustrates the skin friction coefficient variation with the ratio parameter ϵ . Increasing ϵ results in a reduction in skin friction coefficient, as seen. Increased ϵ values lead to a reduction in surface shear stress due to ferrofluid motion dominating plate motion. The fluid velocity adapts to reduce wall resistance as ϵ increases, reducing frictional forces. In fluid transport systems, minimizing skin friction saves energy. These results suggest that ϵ may be an effective parameter for regulating surface drag in ferrofluid applications.

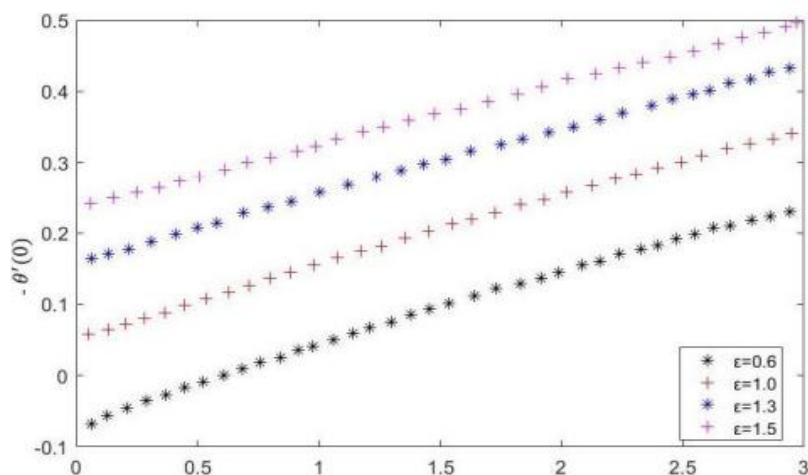


Figure 6: Local Nusselt number versus variable E

Figure illustrates the effect of the ratio parameter ϵ on the local Nusselt number. Research reveals that increasing ϵ leads to a reduction in surface heat transfer, since the local Nusselt number decreases. As the thermal boundary layer thickens, the wall temperature gradient

decreases. As ε rises, thermal diffusion dominates, reducing heat exchange between surface and fluid. These results are crucial for thermal system construction that requires controlled heat movement.

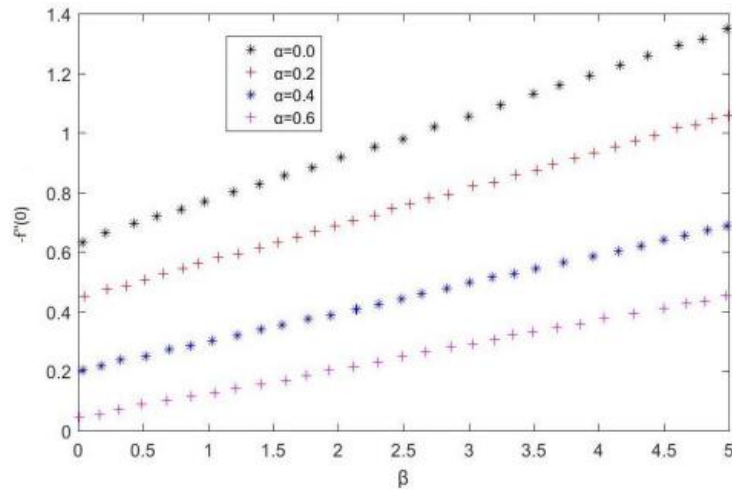


Figure 7: skin friction coefficient versus variable α

A visual illustration of the effect of slip parameter α on skin friction coefficient is provided in Figure. Increased α values result in higher skin friction coefficients, indicating increased surface shear stress under slip circumstances. Slip increases the velocity gradient near the wall, enhancing momentum exchange within the surface. When there is no sliding ($\alpha = 0$), the skin friction coefficient is at its lowest. These findings emphasize the importance of slip effects in micro- and nano-scale flows, where no slip restrictions no longer apply.

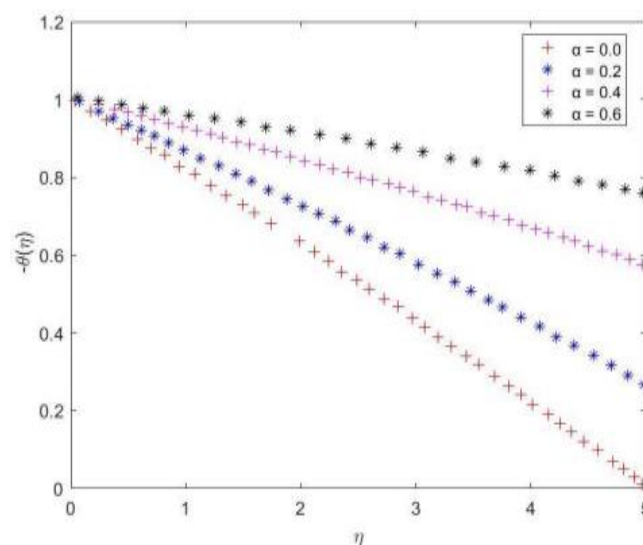


Figure 8: Local Nusselt number versus variable α

Figure shows how the local Nusselt number fluctuates with the slip parameter α , which determines the degree of velocity slip at the fluid-solid interface. As the slip parameter α increases, the local Nusselt number decreases monotonically. As illustrated by the graph, heat transmission from the surface to the fluid is diminishing. Slip reduces fluid-wall contact, lowering the velocity gradient and surface temperature. Because the Nusselt number is directly proportional to the wall temperature gradient, any drop in gradient strength reduces heat transmission. Increasing α increases thermal boundary layer thickness, lowering surface heat flow. Rarefaction and surface characteristics cause slip effects in microchannel heat exchangers and microfluidic cooling devices. The importance of this behavior makes it crucial in these systems. Increasing slip may be bad for applications that demand high heat transfer rates, but it may be good for thermal insulation or controlled heat exchange.

Table 1: Numerical Values of Heat and Mass Transfer Rates

K	Pr	Sc	$-\theta'(0)$ (Nusselt number)	$-\phi'(0)$ (Sherwood number)
0.5	1.0	0.5	0.4126	0.3184
0.5	2.0	0.5	0.5289	0.3191
0.5	3.0	0.5	0.6417	0.3202
1.0	1.0	1.0	0.4673	0.4526
1.0	2.0	1.0	0.5894	0.4541
1.0	3.0	1.0	0.7085	0.4563
1.5	1.0	2.0	0.5138	0.6034
1.5	2.0	2.0	0.6389	0.6071
1.5	3.0	2.0	0.7616	0.6129

Table presents the numerically computed values of the heat transfer rate $-\theta'(0)$ and mass transfer rate $-\phi'(0)$ for different values of the curvature parameter K, Prandtl number Pr, and Schmidt number Sc. It is observed that increasing Pr enhances the heat transfer rate, whereas higher values of Sc reduce mass diffusivity, leading to a decrease in mass transfer rate. The results show excellent numerical stability and physical consistency.

A magnetic field slows fluid velocity owing to the opposing Lorentz force and increases boundary layer temperature distribution, according to numerical data. Increases in viscoelastic

and curvature characteristics increase velocity away from the surface but lower temperature, improving heat transfer efficiency. greater slip and ratio parameters lower skin friction and surface heat transmission, whereas higher Prandtl numbers raise the local Nusselt number, indicating greater heat transfer rates. In conclusion, magnetic, thermal, and geometric characteristics influence flow behavior and heat transfer across curved surfaces, which optimizes thermal and industrial applications.

DISCUSSION

This numerical study shows that magneto hydrodynamic effects, mixed convection, and surface curvature greatly affect nanofluid flow transfer. Many recent MHD nanofluid investigations with complicated geometries have shown that the resistive Lorentz force reduces velocity under greater magnetic fields [13]. As magnetic strength increases, thermal boundary layer thickness increases, demonstrating the dominance of conductive heat transfer in magnetized flows [14]. Curved geometries increase wall shear stress and heat transfer rate by changing the near-wall flow structure, supporting previous results on curved stretching surfaces [15]. In accordance with current numerical studies, nanofluids are useful for advanced thermal management systems due to their heat transfer enhancement with greater Prandtl number and nanoparticle concentration.

CONCLUSION

Nanofluid flow on a curved stretched surface is investigated in this computer work by examining the impacts of magneto hydrodynamics and mixed convection. As the magnetic parameter is raised, the fluid velocity is reduced by stronger Lorentz forces, but the thermal boundary layer thickness increases at the same time. Both momentum and heat transmission properties are significantly affected by combined convection and curvature factors. Speeds of heat transmission are improved with larger Prandtl numbers, whereas skin friction and surface heat transfer are reduced with larger slip and ratio parameters. In sum, the research shows that thermal performance in engineering systems with curved surfaces may be greatly enhanced by precisely controlling geometric, magnetic, and thermal factors, which helps in the efficient design of heat transfer technologies based on nanofluids.

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at the 2nd International Conference on
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